FEATURES OF THE DEEP APPROACH TO MATHEMATICS LEARNING: EVIDENCE FROM EXCEPTIONAL STUDENTS

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It is widely acknowledged that there are individual differences in the way students approach the learning process, and that these are reflected in the learning outcomes. Little research has been done from the learning approaches perspective regarding mathematics learning. We report an exploratory study investigating the features of the deep approach to mathematics learning. We present the case study of two exceptionally competent students who participated in an in-depth interview. Indicators of the deep learning approach along the categories Goals, Study/Learning strategies, Self-regulation aspects, and Motivation are presented. These findings can be employed in the design of instruments to be used in quantitative research.

THEORETICAL BACKGROUND

It is widely acknowledged that there are individual differences in the way students approach the learning process. A main distinction is the one of the superficial versus the deep approach to learning (Entwistle & McCune, 2004). The surface approach is associated with the intention to reproduce the content when necessary. On the other hand, the deep approach to learning is associated with the intention to understand, and is typically related to stronger conceptual understanding of the intended material as well as with higher performance (Chin & Brown, 2000; Chiu, 2011; Smith & Wood, 2000; Stathopoulou & Vosniadou, 2007).

An important question in this research area is the description of the features of each learning approach and their indicators (Cano & Berbén, 2009; Entwistle & Cune, 2004). This is particularly the case for the deep approach to learning, for which different researchers opt for different features and/or indicators. Moreover, empirical studies validating features and indicators have been conducted mainly with adult participants (typically university students). As Entwistle and Cune (2004) argue, however, the defining features of each learning approach cannot be generalized across different disciplines and age groups.

Research from the learning approaches perspective is scarce with respect to mathematics learning and focused mainly on tertiary education (e.g., Cano & Berbén, 2009; Smith & Wood, 2000; but see also Chiu, 2011, for a study with late primary school students).

In a previous study (Bempeni & Vamvakoussi, 2015), we attempted to capture the features of the deep and the surface approach to mathematics learning for secondary school students. Following Stathopoulou and Vosniadou's (2007) work on learning approaches to science learning with adolescent participants, we started with three categories, namely Goals, Study strategies and Awareness of understanding. Our results showed that students with surface approach value school performance as a goal adopt memorization and rehearsal as study strategies and have low awareness of their understanding and of the effectiveness of their study strategies. On the contrary, students that follow the deep approach combine theory studying and extensive practice, invest time in mathematics studying on a long-term basis, and are highly aware of their understanding and of the effectiveness of their study strategies. However, our data indicated that there are other aspects of our participants' approach to mathematics learning that were not captured by our initial categories, mainly regarding motivational and self-regulatory aspects of mathematics learning and studying (see also Cano & Berbén, 2009; Entwistle, McCune, & Tait, 2013).

In the present study we attempted to detect and describe in greater detail the features of the deep approach in mathematics learning by studying exceptionally competent students in mathematics. We adapted appropriately our previous instrument and we enriched it with the categories Motivation and Self-regulation.

METHODOLOGY

Participants

The participants of the study were two students, one sixth grader (hereafter S_1) and one ninth grader (hereafter S_2) with exceptional competence in mathematics according to their mathematics teachers. We note that we didn't rely merely on this information; we also tested their conceptual knowledge in a specific content area, namely rational numbers.

Specifically, we used 25 tasks compatible with Rittle-Johnson and Schneider's (2014) categorization of tasks targeting mathematical conceptual knowledge: a) evaluate unfamiliar procedures, b) evaluate examples on concept, c) evaluate quality of answers given by others, d) translate quantities between representational systems, e) compare quantities, f) invent principle-based shortcut procedures, g) generate or select definitions of concepts, h) explain why procedures work.

These students dealt with these rather challenging tasks very successfully, indicating that they had deep conceptual knowledge in this content area, and also that they were highly competent in mathematical reasoning, in problem solving, and in explaining and justifying their reasoning.

Research instruments

We developed 29 items in the form of scenarios that the students had to react to (e.g., "*If you had to advise a younger student how to study mathematics what would you consider important to tell?", "You observe a friend of yours studying mathematics without solving exercises. You see him dedicate a lot of time studying the theory, making diagrams, going back to previous units, taking notes. Do you study mathematics in the same way? Is there any advice that you would like to offer?",* *"A younger student asks for your help with the comparison of fractions. What would you do to help him?*").

Procedure

The students participated in two in-depth semi-structured individual interviews. During the first interview, they were asked to solve the rational number tasks, thinking aloud and explaining their answers. During the second interview students were asked about their learning approach to mathematics. The second interview took place about three days later. Each interview lasted about one and half hours. All interviews were recorded and transcribed.

Data analysis

The starting points for our analysis were the following categories: a) Goals, b) Study strategies, c) Awareness, d) Self-Regulation, and e) Motivation. The indicators for each of the categories were: a) Understanding-Personal making of meaning, b) Combining theory and practice, Validation, Long-term time investment, Integration of ideas, c) High, d) Monitoring, Regulation, Control of cognition and emotions, e) Intellectual challenge, respectively (Bempeni & Vamvakoussi, 2015; Entwistle et al., 2013).

Features of the deep approach in the learning of mathematics	
Categories	Indicators
Goals	Understanding - Personal making of meaning
	Academic success
Study/Learning strategies	Active involvement
	Validation
	Combining theory and practice
	Long-term time investment – Solving unfamiliar problems
	Integration of ideas
Self-regulation aspects	Monitoring/regulation of understanding
	Awareness of the understanding and the effectiveness of one's strategies
	Regulation of emotions during the exam
	Regulation of study behaviour
	Flexibility in the use of strategies
Motivation	Intellectual challenge

Table 1: Features of deep approach in the learning of mathematics

We selected sentences as unit analysis, but in some cases we used paragraphs so as to obtain a sense of the whole. We looked for utterances that included keywords pertaining to the indicators of each category (e.g., understand, concept, meaning for the indicator personal construction of meaning). We placed the sentences in the coding categories according to the initial indicators and developed new indicators when needed. After coding, data that could not be coded were identified and analyzed later to determine if they represented a new category. New indicators emerged for the categories Goals, Study strategies, Self-regulation and Motivation.

In addition, we merged the categories Awareness and Self-regulation in one more general category namely Self-regulation aspects because in our data utterances related to awareness and self-regulation typically were intertwined. The categories are presented in Table 1.

RESULTS

Goals

Both students stressed that they care about marks, and also for their teachers' and schoolmates' opinion. However, they also stressed the importance of understanding in mathematics and especially of personal making of meaning.

> [Mathematics] is not rote learning. The point is to try on our own and understand. If I could not cope with mathematics and the teacher graded me higher than I deserved, I would try more. Mathematics is a useful subject and I have to understand it. [...] Fractions do not only relate to comparison rules. First of all, you must understand what fraction is. If you do have everything in your mind and know what a fraction represents, then it is easier to solve what you are asked and to consider fractions much more familiar. (S_1)

> Mathematics is not like other subjects that you have to memorize things-you must put your mind to the work, think sensibly. I prefer discovering new things on my own, because in that case I will never forget them. [...] There are other ways to compare fractions except rules. You do have to understand the fraction as quantity, to represent it with a figure. Estimating, using common sense... (S_2)

Study strategies

Active involvement. Both students indicated that they are actively engaged in learning in the mathematics classroom: they recognize what they do not understand, do not hesitate to express their questions and assess the information they receive.

> [A good student] is not reluctant to express and support his/her opinion. […] When I don't agree with my teacher I always step up. For example, I could not understand why we cannot use decimals as fraction terms. Since the fraction represents a division, why is it not allowed to use decimals as numerators or denominators? Decimals can also be divided, can't they? (S_1)

> Once I had doubts about what my teacher said. But I dared to express my objection and we had a scientific debate. I gave it up only when I realized that I was wrong. However, sometimes I happen to be right. (S_2)

Validation. As we can conclude from the above mentioned transcripts, the two students are not willing to accept something if it is not sufficiently proved. At different points of the interview, they mentioned that they use «common sense» to check their results or to monitor their steps while solving (see also transcripts in the section «Self-regulation aspects»). We note that both students monitored the solution process during the first phase of the study (e.g., they used counterfactual proof).

Combining theory and practice. Both students referred to the importance of combining deep understanding of theory and solving exercises.

> Mathematics is theory too, if you don't understand the theory well you cannot solve problems. There is always some theory behide the problem. How can you solve a problem with proportions if you have no idea of what proportion is? You must also practice with many exercises. But learning the rules by heart does not help. Then, in problems, how can the rules be useful to me? Will I simply write down the rule? (S_1)

> Both theory and exercises are important. If you do not study theory, you cannot solve but only the simplest problems. (S_2)

Long-term time investement – Solving unfamiliar problems. Both children appeared to value the long-term time investment on mathematics studying.

> It is necessary for students to do extensive practice in mathematics, because when gaps are created, it is quite difficult to understand the more advanced material. That is why I try to solve many exercises by myself except the ones I have for homework. (S_1)

> Studying should not be restricted to what is required in the course. […] I do a lot of practice during the private tutoring lessons I attend. (S_2)

For these students, practice is not limited to the study of solved examples or to solving similar problems.

> Solving many similar exercises is not enough. Then if you are asked to solve a slightly different problem, you cannot do it. This is because you can deal only with similar problems, with different numbers. I think that if somebody has not understood the material, then they cannot think further and solve unfamiliar problems. (S_1)

> I do not like solving similar exercises all the time. Repetition may be useful for other subjects but not for mathematics. For example, I do not believe that memorizing the solutions of exercises in mathematics is useful even if one can solve them when asked. (S_2)

Integration of ideas. Both students referred to importance of making connection among different units of mathematics and also relating mathematics to other subjects (Physics, Chemistry), and to everyday life, too.

> Yes, I think that the previous and the following units are connected in some way in mathematics. For example, we had been taught proportions and then percents, for which good knowledge of proportions was necessary. And if you want to understand proportions well, you need to understand fractions as well. (S_1)

> We were taught the distributive property with numbers when we were at sixth grade, and then we were taught the same property with variables, and the same holds for all other properties. (S_2)

It is worth mentioning that both students valued the connection of different representations in mathematics, and also to everyday life as an appropriate instructional method.

Teachers need to make mathematics real for students, to show mathematics in real life. For example when we say ¼ kg cheese what do we mean? How much is it? (S_1)

In the first years of school-life students have not understood fraction as quantity. I could help a younger student to understand it with figures and representations. (S_2)

Self-regulation aspects. Both students appeared to monitor, control and regulate themselves in the level of cognition, emotions and behavior in the learning of mathematics.

When I face a difficulty, I try to see the problem from many different aspects and construct a table with what is given and asked in my mind. You can be aware if the process goes well while solving, if you monitor what you're doing and do not solve it mechanically. You can also verify by putting numbers in case you want to make sure that you are correct. I validate in my mind without making operations. You should also pay attention to the result, the result should be reasonable. (S_1)

I am sure that I have understood the problem, when I am able to put that in my own words, when I have the problem in my mind and it is not necessary to read it all the time. When I have difficulty in understanding the problem I break it into small parts and then I try slowly to understand what I do not do well. (S_2)

As a result, both students appeared to have a high awareness of understanding and to be able to differentiate the difficulties in understanding from the school requirements.

At first, I found fractions a little bit difficult. Not the operations and the exercises, these were very easy. (S_1)

I did not understand the unit "probability" that we were recently taught. I found it disjointed but I tried to understand using paper and pencil. […] Understanding the concepts that you are now taught in a greater grade is something very usual. For example we are accustomed to «cross-multiply». But you should look into it deeper, so as to understand the algorithm. (S_2)

What is also notable is their reference to the way they face an unfamiliar problem in the exam context.

You have to try until the last moment. If you make negative thoughts from the very start, then you will not solve the problem even if you possess the sufficient knowledge to do it. If you have time, you can try until the end. There is no reason to give up. (S_1)

At first you say, "Oh my God", then you are starting to swear, and finally you say "I will do my best. I will not die, after all, it's just a test! (S_2)

Both students appeared to recognize that the combination of insistence on trying and flexibility is necessary.

Once I had difficulty with a problem in a test I left it last. When I came back to it, I tried to look it from another perspective. Generally, when I realize that my method is not efficient, I try to apply some other knowledge, even if I am not sure that this is the correct way to solve the problem. (S_1)

> I simply made different thoughts. And when my thoughts took me nowhere, I rejected them. I thought different things regarding the solution and I got rid of the ones that did not help me. (S_2)

We note that S_1 stated that she is always concentrated when studying so as to need less time. S_2 «revealed» that he started courses with a personal tutor because he wanted somebody to motivate him to do more practice. Both students showed self-confidence regarding their current learning strategies in mathematics. S_2 mentioned that he did not pay attention to the theory in the past and he added: «I realized it later, but I do not believe it was late». Moreover, he referred to his strategy focusing more on exercises and stated: «I understand in my own way. If I realize that this way is inappropriate, I will change it».

Motivation. Both students appeared to be motivated by unfamiliar and challenging problems.

> I prefer problems that are difficult, when you need to think of something by yourself. I don't like the ones that are solved in a particular way, mechanically. I find all these exercises with tables that we do the method of cross-multiplying all the time very boring. (S_1)

> I find uninteresting what keeps me from going further. Everything that has operations and you must do constantly the same. That's why Geometry is a more interesting part to deal with. (S_2)

CONCLUSIONS-DISCUSSION

This exploratory study investigated the learning approach to mathematics of exceptionally competent students, with the intention to trace features of the deep approach to mathematics learning. The results provide indicators along the categories Goals, Study/Learning strategies, Self-regulation aspects and Motivation (Bempeni & Vamvakoussi, 2015; Entwistle et al., 2013).

More specifically, the two students value the personal making of meaning, without neglecting academic success. They invest time in the study of mathematics, and consider the solving of unfamiliar problems an important part of practice. Despite the fact that they recognize the value of the theory, they do not dedicate much time to study it. This inconsistency may be explained by the quality of participation in the school classroom which is a central learning strategy for these students. They also actively look for connections among different representations, content units, different subjects, and everyday life. Validation of mathematical knowledge is highly significant for them: they actively seek for validation in the school context and when they solve problems. Furthermore, both students monitor, regulate, and control their emotions and their behavior in the context of mathematics learning and studying. As a result, they are highly aware of their understanding and their learning strategies, and they are flexible. Finally, they are motivated by intellectual challenge.

The findings of the present study offer a more detailed insight into the features of the deep approach to mathematics learning and can form the basis for the design of a research instrument to be used in quantitative studies. It should be noted, however, that these findings also point to the fact that the construct "learning approach" is rather broad and overlaps with other constructs stemming for other research perspectives (e.g., "intentional learning", "self-regulated learning" – for a similar observation see Cano & Berbén, 2009). More detailed analysis of such constructs is necessary to highlight possible similarities and differences.

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