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Conceptual and procedural strategies in rational number tasks and their relation to ninth graders' approaches to the study of mathematics

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Abstract

In the present study we examined ninth graders' procedural and conceptual strategies in rational number tasks and their relationship to students' approach (deep/superficial) to the study of mathematics. We found individual differences-sometimes extreme-in the way that students combine procedural and conceptual knowledge for rational numbers. Moreover, it appears that these differences may be associated with differences in students' approaches to studying. Specifically, our findings indicate that students who consistently use only procedural strategies adopt superficial approaches to studying, whereas students who consistently use conceptual strategies adopt deep approaches to studying.

Theoretical background

The term "number sense" was coined to describe deep conceptual understanding of numbers that goes beyond mere procedural competence and the ability to use this understanding in flexible ways (e.g., McIntosh, Reys, & Reys, 1992; Verschaffel & De Corte, 1996).

The distinction between rational number sense procedural and conceptual components, is grounded on the distinction between procedural and conceptual knowledge. Procedural knowledge is defined as the ability to execute action sequences to solve problems. On the other hand, conceptual knowledge is defined as the implicit or explicit understanding of the principles that govern a domain and of the interrelations between knowledge units in the domain (Rittle-Johnson, Siegler, & Alibali, 2001). There is a lot of discussion regarding which type of knowledge develops first and there is evidence in favour of contradictory views (Rittle-Johhnson et al., 2001).

Hallett, Nunes and Bryant (2010) suggested that a possible explanation for the contradictory findings is that not much attention is paid to the individual differences in the way students combine the two kinds of knowledge.

Schneider and Stern (2010) highlighted another methodological issue: Typically it is assumed that conceptual and procedural components of knowledge in a domain (e.g., rational number sense) can be measured independently of each other via the use of appropriate tasks. It is possible, however, that solutions of conceptual assessment tasks might, to some degree, also reflect procedural knowledge and vice versa. Moreover, for tasks administered in paper-and-pencil tests, it is often impossible to decide whether the student used an algorithm or not.

In the present study, we assessed ninth graders' number rational sense. We used tasks assessing conceptual and procedural aspects of rational number sense as reported in the relevant literature (e.g. McIntosh et al. 1992). However, taking into account the methodological difficulties mentioned above, we designed a qualitative study in order to investigate the kind of strategies that the individual student uses, regardless of the procedural/conceptual nature of the task. Again, we relied on relevant literature to identify procedural and conceptual strategies (Yang, Reys, & Reys, 2007). Procedural strategies are related to rules and exact computation algorithms whereas conceptual strategies rely on conceptual components of rational number sense.

Similarly to Hallet et al. (2010), we hypothesized that there are individual differences in the way students combine the two kinds of knowledge. We predicted that there are extremely different profiles, namely students that consistently follow procedural approaches and fail in conceptual tasks and students with poor procedural knowledge that successfully apply conceptual strategies.

In addition, we attempted to investigate possible reasons underlying the predicted individual differences. One such reason could be students' mathematics-related beliefs (Op 't Eynde, De Corte, & Verschaffel, 2002). We reasoned that students who view mathematics as a set of rules and procedures to be followed are more likely to belong to a procedural profile than students who view mathematics as a body of notions and procedures that are meaningfully interrelated. We adopted Stathopoulou and Vosniadou's (2007) model that predicts the indirect influence of beliefs on student's understandings via their approach (deep/superficial) to studying. A deep approach to studying involves goals of personal making of meaning, deep strategy use (e.g., integration of ideas), and awareness of understanding or the lack of it. A superficial approach involves performance goals, superficial strategy use (e.g., rote learning), and low awareness of understanding. We hypothesized that procedural profile students follow superficial approaches to studying, whereas conceptual profile students follow deep approaches.

Methodology

Participants

The participants were seven students at grade nine from schools in the area of Athens. By grade seven, they had been taught all the material related to rational numbers, including fractions and decimals operations, equivalent fractions, comparing and ordering rational numbers, turning a fraction to decimal and vice versa. They had also encountered many problems pertaining to various aspects of rational numbers and had been exposed to various representations, including the number line.

Materials

We used nineteen rational number sense tasks grouped in three categories. The tasks of the first category (e.g., ordering) could be solved by standard procedures taught at school. The second category included tasks targeting conceptual knowledge, such as translating between representations, and estimating the magnitude of rational numbers. The tasks of the third category required deep conceptual understanding or the combination of conceptual understanding and procedural fluency. For example, there were tasks targeting the dense ordering of rational numbers and problems that require the understanding of rational numbers as quantitative relations.

In addition, we developed fourteen items so as to investigate the kind of approach (deep/superficial) students adopt in the study of rational numbers as well as of mathematics in general. The items were presented as scenarios describing a situation that the student had to react to (e.g. "*A younger student asks for your help with the comparison of fractions. What would you do to help him?*").

Procedure

Each student participated in two individual interviews. In the first, the students dealt with the rational number sense tasks. They were asked to think aloud and explain how they reached their answers. About a week later, they were interviewed about their study approaches. Additionally, they were asked to comment on the responses of the first questionnaire (certainty about the solution, awareness of their performance in tasks). All interviews were recorded and transcribed.

Results

Based on students' responses in all tasks three student profiles were defined. As shown in Table 1, the students in the first profile dealt successfully with all tasks, moving flexibly between procedural and conceptual strategies. The students in the second profile responded correctly to all tasks that could be solved via the application of procedural strategies, but failed when this was not the case. Only one student (hereafter, S_1) was placed in the third profile. S_1 used consistently conceptual strategies, sometimes rather advanced, such as such as the use of sophisticated representations. For instance, when asked *«how many numbers are there between 2/5 and 3/5»,* she responded *«If we locate them on the number-line a gap is created. In this gap, there are infinitely many numbers».* However, S_1 failed in all tasks that required procedural fluency (e.g., computations).

Table 1 Students' performance and kind of strategies used in rational number sense tasks

	Performance		
Rational number sense tasks	Profile A (n=3) Procedural-Conceptual	Profile B (n=3) Procedural	Profile C (n=1) Conceptual
Tasks assessing procedural aspects	Success	Success	Failure
Tasks assessing conceptual aspects	Success	Failure	Success
	Kind of strategies		
Tasks that could be solved with paper-and-pencil	Procedural/ Conceptual (Selection of the efficient strategy)	Procedural	Conceptual

For the second phase of the study, we present results for two students, namely S_1 and one student from group c (hereafter, S_2), who had similar school grades. S_2 failed in all tasks that presupposed conceptual knowledge. For example, she estimated that 3/8 is closer to 1/2 than 7/13 explaining that *«the denominator 8 is closer to 2»*.

The criteria for identifying S_1 and S_2' s studying approaches were defined in terms of their goals, strategies, and awareness of understanding (Stathopoulou & Vosniadou, 2007). The analysis of their responses to all items showed that S_1 adopted a deep, whereas S_2 a superficial approach to studying.

Specifically, S_1 valued personal meaning making as a goal (e.g., "*It is important to make sense of what you study in mathematics*", "*If you understand the meaning of fraction then you can compare fractions*"). On the other hand, S_2 appeared school-performance oriented (e.g., "*What I would* *advise a younger student is to focus on what is likely to be asked in the exams*").

S_1 valued active involvement and strategies aiming at integration of ideas and in (e.g. "*Mathematics is not about rote learning, you have to put your mind to the work*", "*It is important to solve unfamiliar problems on your own*"). On the contrary, S_2 repeatedly referred to the importance of memorization and described her study habits with expressions such as the following: "*I look at what we have done at school and the exercises, so that I remember how to solve them*").

Finally, S_1 detected all the rational number sense tasks that she had answered incorrectly and was aware of the fact that she lacked procedural fluency. On the contrary, S_2 was confident that she had answered all the rational number sense tasks correctly and that she had a firm understanding of rational numbers in general.

Discussion-Conclusions

The results presented support our hypothesis that there are individual differences, even extreme, in the way that students combine conceptual and procedural knowledge of rational numbers. Our findings also indicate that differences in the level of rational number sense development may be associated with differences in approaches to studying. This possible relationship, as well as its connection to students' mathematics-related beliefs, needs to be further investigated.

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