THE APPROACH TO MATHEMATICS LEARNING AS A PREDICTOR OF INDIVIDUAL DIFFERENCES IN CONCEPTUAL AND PROCEDURAL FRACTION KNOWLEDGE

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In the present study, we tested the hypotheses that: a) there are individual differences in secondary students' conceptual and procedural fraction knowledge, and b) these differences are predicted by students' approach (deep vs. surface) to mathematics learning. We used two instruments developed and evaluated for the purposes of the study which were administered to 463 students at seventh and ninth grade. We found four clusters of students corresponding to different ways of combining conceptual and procedural knowledge of fractions. Students' approach to mathematics learning predicted membership to some, but not all clusters.

THEORETICAL BACKROUND

Procedural knowledge is commonly defined as the knowledge of algorithmic procedures, whereas conceptual knowledge as the knowledge of concepts and principles pertaining to a certain domain (Rittle-Johnson & Schneider, 2015). This distinction has been criticized (e.g., Star & Stylianides, 2013), a main issue of concern being whether it is possible for the two types of knowledge be separated, given that they are typically found to be highly correlated. Nevertheless, there are indications that the two types of knowledge can be separated both theoretically and empirically (Lenz & Wittman, 2021), and this distinction remains useful in the area of research on mathematics learning (Vamvakoussi, Bempeni, Poulopoulou, & Tsiplaki, 2019).

Assuming that conceptual and procedural knowledge are distinct types of knowledge, the order of acquisition and their relation have long been an issue of interest. The currently predominant theory, namely the iterative model (Rittle-Johnson, Siegler, & Alibali, 2001), came to bridge the gap between two different accounts according to which one type of knowledge precedes the other (procedures-first and concept-first theories). The iterative model assumes that either type of knowledge can trigger the learning process, depending on the child's prior experience with the domain in question; and that, from then on, the links between the two types of knowledge are bi-directional and continuous, with increases in one kind of knowledge leading to gains in the other type of knowledge. The iterative model explains many empirical findings, notably the well-established one that the two types of knowledge are positively correlated. However, such correlations found at group level do not accurately depict

2021. In Inprasitha, M., Changsri, N., Boonsena, N. (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, pp. 57-64. Khon Kaen, Thailand: PME.

what happens at the individual level (Vamvakoussi, et al., 2019). Indeed, there is evidence that there are individual differences in the ways students combine the two types of knowledge. Hallett and colleagues (2010; 2012) investigated such individual differences in the area of fraction learning and identified different groups of students (Grades 5-8) with the one type of knowledge, conceptual or procedural, to be more developed than expected, given the other type. Similar individual differences in fraction knowledge have been found for older students, namely 9th graders (Bempeni, Poulopoulou, Tsiplaki, & Vamvakoussi, 2018; Lenz & Wittman, 2021), and they may be even extreme (Bempeni & Vamvakoussi, 2015).

With the aim of explaining how these individual differences regarding knowledge in the domain of fractions, or other domains, arise, several hypotheses have been tested looking at various factors such as the amount of the prior knowledge in a domain (Schneider, Rittle-Johnson, & Star, 2011); differences in cognitive profiles, measured as general conceptual and procedural ability (Gilmore & Bryant, 2006; Hallett et al., 2012) or general cognitive abilities (Lenz & Wittman, 2021); and differences in educational experiences, measured as attendance in different schools or as school grade (Canobi, 2004; Hallett et al., 2012). No or limited support for these hypotheses has been found.

We have formulated the hypothesis that a possible source of individual differences in conceptual and procedural fraction knowledge is the individual's approach to mathematics learning. In the literature there is an overarching distinction between the deep approach to learning, associated with the individual's intention to understand; and the surface approach, associated with the individual's intention to reproduce. There are several ways of characterizing each approach, mainly adapted to tertiary education (Entwistle & McCune, 2004). In a qualitative study (Bempeni & Vamvakoussi, 2015) we adopted a model developed by Stathopoulou and Vosniadou (2007) and tested with secondary students. This model differentiates between the deep and the surface approach to learning along three axes, namely goals (personal making of meaning vs. performance goals); study strategies (e.g., searching for connections vs rote learning); and awareness of understanding (high vs. low). We interviewed in depth three 9th graders (A, B, C) who differed with respect to their fraction knowledge: A had strong conceptual as well as procedural knowledge; B had strong conceptual, but extremely weak procedural knowledge; and C had strong procedural, but extremely weak conceptual knowledge. We found indicators of the deep approach to mathematics learning for A and B, and indicators of the surface approach for C. We also traced differences among the students with respect to particular aspects of motivation (e.g., enjoying vs. avoiding intellectual challenges in mathematics). In a second qualitative study, we further investigated the features of the deep approach to mathematics learning by studying exceptionally competent students in mathematics (Bempeni, Kaldrimidou, & Vamvakoussi, 2016).

These two qualitative studies, informant the development of an instrument assessing secondary students' approach to mathematics learning (deep vs. surface) along four axes, namely goals, study strategies, motivation, and self-regulatory behaviors (e.g., monitoring of understanding, regulation of study habits).

In the present study, we examined the hypotheses that there are individual differences in conceptual and procedural knowledge of fractions (hereafter, CKn and PKn) that become less salient but remain present up to Grade 9; and that these differences are predicted by students' approach to mathematics learning (surface vs. deep).

METHOD

Participants

The study had two phases. The participants in the first phase were 510 students at Grades 7 and 9, of whom 463 participated also in the second phase (262 ninth graders and 201 seventh graders). The participants came from seven Greek secondary schools.

Materials

Students' CKn and PKn was measured by an instrument that has been evaluated in a previous study with respect to reliability and validity (Bempeni et al., 2018). The instrument comprised 12 procedural tasks (e.g.: fraction operations, simplification of a complex fraction) and 14 conceptual tasks such as fraction representation, comparison, estimating the outcome of fraction operations (see Bempeni et al., 2018; Vamvakoussi et al., 2019 for a more detailed description of the instrument).

The new instrument assessing student's approach to mathematics learning comprised of 28 statements and 6 scenarios in which two hypothetical students presented two different views on an issue. Half of the statements were consistent with the deep approach to learning, and the other half with the superficial approach to learning. The students were asked to express the degree of their accordance in a scale of 1-4 (1=Totally Disagree, 2=Disagree, 3=Agree, 4=Totally Agree). The neutral choice "Neither Agree or Disagree" was not included because it has been proved problematic in similar studies (e.g.: Entwistle et al., 2015). Examples of such statements were the following: *"It's a waste of time to study for something that is not required for the exams", "If I do not remember the particular strategy to solve a problem, it is meaningless to try to solve it", "I prefer to solve new problems, than practicing with the ones I already know how to solve".*

Procedure

The students had fifty minutes to complete the first questionnaire with the fraction tasks, which was enough for them. The questionnaire for the approach to mathematics studying and learning was administered three weeks later. No time limit was imposed, but the students needed at about half hour to complete it.

DATA ANALYSIS – RESULTS

1st Phase of the study

The data of the first phase of the study were classified using the proposed hierarchical method of cluster analysis, and taking as variables the standardized residuals in the two types of tasks (Bempeni et al., 2018; Hallett et al., 2010, 2012). By following this method, we examined the *relative* difference between the two variables. Using a series of evaluation measures in R programming language (R project for statistical computing), we determined that the optimal number of clusters was 4.

In Figure 1, we present the average performance in conceptual and procedural knowledge by cluster. In a little more detail, the first cluster (*"Stronger than expected in CKn and PKn",* N=163, 32%, 10% 7th Grade) performed better than expected in both types of tasks. The second cluster performed better than expected in procedural tasks based on their CKn (*"Stronger than expected in PKn",* N=207, 40.6%, 28.6% 7th Grade). The third cluster performed better than expected in conceptual tasks based on their PKn (*"Stronger than expected in CKn"*, N=75, 14.7%, 6.9% 7th Grade). Finally, the fourth cluster (*"Weaker than expected in CKn and PKn"*, N=65, 12.7%, 8.4% 7th Grade), comprised of students with low performance in both measures. It is worth noting that despite the fact that the overall score of the cluster *"Stronger than expected in PKn"* was higher than the one of the cluster *"Stronger than expected in CKn"*, the CKn score was comparatively lower. Moreover, the average performance in PKn and CKn was better at 9th grade (69.5% and 49.2% respectively) than at 7th grade (66.9% and 32.8%).

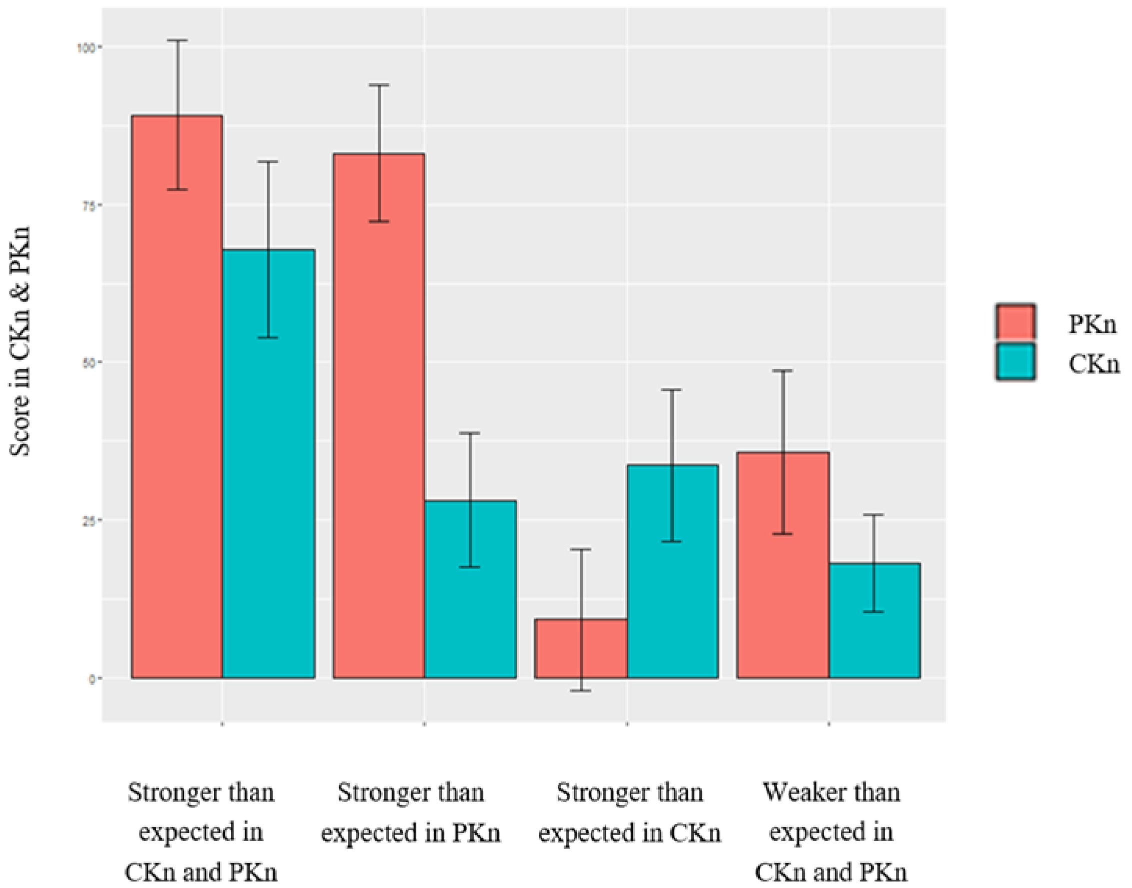

Figure 1: Average performance in CKn and PKn by cluster

2nd Phase of the study

In the second questionnaire, for the items consistent with the deep approach to learning, each choice (1-4) was taken to reflect the degree (low to high) of consistency of the response with the deep approach to learning. For the items consistent with the surface approach to learning the scores were ranked in the inverse order. The total score (hereafter, LA score) was calculated as the sum of the scores of all the items. For the analysis of the data, we used R programming language.

For the evaluation of the second questionnaire, we conducted a small pilot study. The participants of the pilot study were 120 seventh and ninth graders. In order to assess the internal consistency of the instrument, we calculated Cronbach's alpha. The value of Cronbach's alpha for two of the items had negative correlation with the scale, and as a result, these questions were excluded from our instrument. Finally, the value of Cronbach's alpha for the scale was $\alpha=0.821$. We also assessed the external consistency of the instrument over a period of 15 days with a test-retest method. Forty-one students completed the questionnaire for a second time. We calculated the value of intra-class correlation coefficient for each item separately. Five of the items displayed intra-class correlation below 0.4 and thus we decided to exclude them from the final version of the instrument.

	Clusters	N	Mean	SD	Median	Range
1	*Stronger than expected in CKn and PKn*	158	2.987	0.414	3.037	(1.852 - 3.704)
2	*Stronger than expected in PKn*	194	2.830	0.397	2.923	(1.630 - 3.593)
3	*Stronger than expected in CKn*	52	2.636	0.275	2.633	(2.222 - 3.370)
4	*Weaker than expected in CKn and PKn*	59	2.593	0.367	2.630	(1.481 - 3.481)

Table 1: Mean LA score by cluster

The test of independence showed that there is a statistically significant correlation between cluster and approach to mathematics studying and learning ($\chi^2=60.396$, df=3, p-value<0.0001). As illustrated in the Table 1, the cluster *"Stronger than expected in CKn and PKn"* had the highest score with respect to the approach to mathematics learning, followed by the group *"Stronger than expected in PKn"*. The group *"Weaker than expected in CKn and PKn"* had the lowest score.

In order to test the hypothesis that learning approach and school grade are predictors of the level of students' CKn and PKn, we conducted multinomial logistic regression (Table 2). The results showed that both learning approach and grade can predict cluster membership. With the cluster "*Weaker than expected in CKn and PKn"* as base level, for every unit that the individual' s LA score increases, it was 21.98 more likely for the student to belong to the cluster *"Stronger than expected in CKn and PKn"* and 4.77 more likely to belong to the cluster *"Stronger than expected in PKn"*. Using the same base level, a ninth grader is 8.35 more likely to belong to the group "*Stronger than expected in CKn and PKn"* than to the group "*Weaker than expected in CKn and PKn"*.

Predictor	*Weaker than expected in CKn and PKn* **Vs.**	B	OR= exp(B)	p-value
Score in mathematics learning approach	*Stronger than expected in CKn and PKn*	3.09	21.98	**0.000**
	Stronger than expected in PKn	1.56	4.77	**0.000**
	Stronger than expected in CKn	0.42	1.53	0.390
9th Grade	*Stronger than expected in CKn and PKn*	2.12	8.35	**0.000**
	Stronger than expected in PKn	0.18	1.19	0.606
	Stronger than expected in CKn	0.52	1.69	0.206

Table 2: Predictive factor testing

CONCLUSIONS – DISCUSSION

The results of our study confirm the hypothesis that there are individual differences in the way students combine CKn and PKn for fractions (Hallett et al., 2010; 2012). Although older students were more likely to have strong CKn as well as PKn, a considerable percentage of 9th graders belonged to the clusters *"Stronger than expected in PKn"* and *"Stronger than expected in CKn"*, indicating that individual differences remain present up to Grade 9. It is worth noting that the greater part of our sample was found in the group *"Stronger than expected in PKn",* indicating that instruction favours mainly the development of PKn (see also Canobi, 2004).

In our attempt to detect the possible factors that are responsible for individual differences in CKn and PKn, we tested the hypothesis that the approach to mathematics learning predicts such individual differences. The LA score predicted the membership in the clusters *"Stronger than expected in CKn and PKn"* and *"Stronger than expected in PKn"*. This result only partially supports our hypothesis, due to the fact that the probability for a student to belong to the cluster *"Stronger than expected in CKn"* cannot be predicted; moreover, the mean LA score for this cluster was the second lowest one, lower than the mean LA score of the "Stronger than expected in

PKn” cluster. A possible explanation is, that as a result of using residualized scores in the cluster analysis (Hallett et al., 2010, 2012; Bempeni et al., 2018), the *“Stronger than expected in CKn”* cluster includes students with *relatively* stronger CKn given their PKn, but not necessarily in absolute terms; and similarly, for students in the *“Stronger than expected in PKn”* cluster. A different method for clustering the students, differentiating between the low from the high performing students could be a viable solution (see Lenz & Wittman, 2021, for such a method).

Whilst the development of the two types of knowledge is not assumed to be symmetrical at any given moment (Rittle-Johnson & Schneider, 2015), our results put a challenge to the iterative model. More specifically, given the age and educational experience of the participants, we would expect a more balanced development of the two types of knowledge which is not the case in our study.

The learning approach to mathematics deserves to be further investigated as a source of individual differences in CKn and PKn. The instrument that we developed is a contribution of some significance per se, since, to the best of our knowledge, there is no similar instrument targeting secondary students. An enrichment and refinement of our instrument, in view of the fact that several items had to be excluded from its final version following its evaluation, is worth-considering.

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