ENHANCING THE DEVELOPMENT OF MULTIPLICATIVE REASONING IN EARLY CHILDHOOD EDUCATION: A CASE STUDY

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We report a case study intervention pilot-testing a program of activities aiming at enhancing pre-primary children's multiplicative reasoning competences. The program treated discrete and continuous quantities in a unified manner; provided learning experiences pertaining to three fundamental multiplicative operations, namely iteration of a quantity, equi-partitioning, and counting with composite units / measuring with fractional units; and introduced terms for multiples and submultiples. Four pre-primary children participated in an intense 4-day intervention. The program of activities was well within their range of abilities and enhanced their competences in terms of their ability to discern and express verbally multiplicative relations; and to tackle multiplicative situations and explain their strategies.

THEORETICAL FRAMEWORK

Research-based evidence indicates that young children perceive, at a rudimentary level, multiplicative/proportional relations (Mix, Huttenlocher, & Levine, 2002), and are able to tackle simple multiplicative situations between discrete as well as continuous quantities (e.g., Hunting & Davis, 1991; Kornillaki & Nunes, 2005). For example, 4-5 year-olds identify pictures of imaginary creatures that are magnified proportionally among others that are not (Sophian, 2000). Provided that a sufficient number of area models of a part-whole relation are presented, 6-year-olds can select a model that represents the same relation albeit via different shapes with respect to kind or size (Goswami, 1989). Children who have not been taught multiplication or division (6-7 years of age) can recognize simple multiplicative transformations of discrete and continuous quantities and predict the effect of the transformation on a different quantity (McCrink & Spelke, 2016). Five to seven-year-olds deduce the principle "more recipients, smaller share" that underlies fair-sharing situations (Kornillaki & Nunes, 2005), for discrete as well as continuous quantities. As could be expected, these early competences manifest themselves in a limited range of contexts and conditions. In addition, there are inter-individual differences with respect to these competences. It is nevertheless important to note that early multiplicative reasoning is enhanced when children are exposed to relevant informal or formal learning experiences (Hunting & Davis, 1991; Van den Heuvel-Panhuizen, & Elia, 2020).

Early education has not capitalized yet on such evidence. For example, an analysis of the latest Greek early mathematics curriculum (K-2) showed that learning objectives pertaining to additive reasoning precede and are far more than the ones for multiplicative reasoning,

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for discrete as well as continuous quantities (Vamvakoussi & Kaldrimidou, 2018). In the Kindergarten curriculum, in particular, all learning objectives for multiplicative reasoning are limited to discrete quantities; no linguistic or other tools for expressing multiplicative relations are mentioned, not even the word "half". Nevertheless, children are intended to familiarize themselves with multiplicative situations pertaining to multiplication, partition, and quotition that call for three fundamental multiplicative operations, namely iteration of a quantity, equi-partitioning, and counting with composite units. These operations are applicable also for continuous quantities, if "counting with composite units" is replaced by "measuring with fractional units". This fact is not exploited in instruction.

These limitations in the Greek kindergarten early math curriculum indicate that early multiplicative reasoning competences are not adequately supported in early instruction, especially in the context of continuous quantity. The lack of terms for multiplicative relations is also important, given that linguistic tools are indispensable for prompting children to attend to the relations embedded in multiplicative situations and recognize the same relation in different contexts (Hunting & Davis, 1991). Indeed, vocabulary pertaining to multiplicative relations in the first grade has been found to uniquely predict proportional reasoning abilities in the second grade (Vanluydt, Supply, Verschaffel, & Van Dooren, 2021).

It could be argued, however, that introducing terms for multiples and submultiples in the first years of instruction as well as extending the multiplicative situations that children are intended to explore to continuous quantities as well, might be beyond the range of abilities of young children.

We designed an program of activities addressing discrete and continuous quantities in a unified manner (see Steffe, 1991, for a relevant recommendation); providing learning experiences pertaining to all three aforementioned operations; and introducing terms for multiples and submultiples. To the best of our knowledge, there is no similar intervention reported in the literature targeting young children. We report results from a case study intervention investigating whether this program a) was within the range of abilities of pre-primary children, b) would enhance children's multiplicative reasoning competences, in terms of their ability to discern and express verbally multiplicative relations; and to tackle multiplicative situations and explain their strategies.

METHOD

The present study is a quasi-experimental case study. (Pre- test/ Intervention/ Post-test without a control group).

Participants

The participants were 4 children (mean age 5 years 7 months, one girl) who had just graduated from kindergarten and were familiar with fair-sharing situations involving discrete quantities (two recipients, no remainder).

Experimental tasks

Pre- and Post-test were conducted via individual interviews. Three tasks were used (A, B, C, 4 trials each) targeting the relations 1:2 and 2:1 for discrete and continuous quantities (represented by concrete materials). Task A was an analogical task where the intended relation (X/Y) was exemplified, with the information that "X matches Y". The children were asked to find the quantity Z matching a new quantity W. Task B was framed as a fair-sharing problem. The children were given the initial quantity and asked to find the share (1:2); and vice versa (2:1). In task C children were explicitly asked to find "half" and "double" of given quantities, and to explain what the terms "half" and "double" mean. Overall, there were 3 trials for 1:2 for discrete quantities, 3 trials for 1:2 for continuous quantities; and similarly for 2:1.

An additional task (D), similar to C, albeit for 1:3/ 3:1 was added in the post-test. The children were given 5 alternatives for each trial and were asked to explain their answers.

Procedure

The children participated in the intervention as a group, during four consequent days (one session per day, about 45΄ each). The pre- and post-test took place one day before and two days after the intervention, respectively. Children and their parents consented to participate in the study. The intervention was carried out by the first author of this paper, a qualified kindergarten teacher.

The intervention

We designed two types of activities. Both types addressed discrete and continuous quantities represented by concrete materials; were embedded in story-based scenarios; and required iteration of a quantity, equi-partitioning, and measuring with different units (composite units for discrete quantity, fractional units for continuous quantity). The first type of activity was based on simple and proportional sharing. The children worked with 24 such multiplicative situations during the first two days. According to the scenario, the children were asked to help imaginary creatures (represented by rectangular bars of cardboard with equal width, but different length) to share candies (discrete) or chocolate bars (continuous) proportionally to their length. The relations between the lengths were 1:1 (fair-sharing), 1:2, and 1:3 (proportional sharing). Depending on what was asked (number of recipients, quota, or the shared quantity), different operations were required.

The second type of activities addressed multiplicative change situations. We employed "fractions machines" (Hunting & Davis, 1991), producing multiples of given quantities from one side (2, 3, 4), and the corresponding submultiples (1/2, 1/3, 1/4) from the other. The children worked with 23 such problems during the last two days.

During the intervention, the researcher modeled the operations and introduced the new terms. The terms for multiples (double, triple, quadruple) were introduced in the context of iteration of a quantity. The children were familiar with the term "half" in fair-sharing situations (equi-partitioning), so the same context was used for the introduction of other terms for submultiples (one third, one fourth). Because equi-partitioning continuous quantities, in particular in three parts, was challenging for the children, fractional pieces of the quantities were available for them to choose from and examine how many times they fit in the given quantities. Thus, the children had to estimate the magnitude of the part first, and then to verify their estimate by measuring the quantity with its part.

RESULTS

Children's response to the intervention

The intervention tasks were challenging for the children. Indeed, most of the tasks were unfamiliar to them, in particular the tasks pertaining to multiplicative change, which was a novel multiplicative situation for them. The children collectively came up with effective strategies for some of the unfamiliar situations (e.g., dealing for fair-sharing to more than two recipients, folding for equi-partitioning a continuous quantity in two parts). More importantly, they appeared capable of adopting and using the intended strategies and vocabulary introduced by the researcher; and to transfer them to the novel situation of multiplicative change. To illustrate this point, we present two episodes that occurred on the fourth day of the intervention. In the first episode, the researcher presents for the first time the children with the 1:3/3:1 "fraction machine" for discrete quantities. The capital letters in the brackets refer to Figure 1, illustrating the use of materials by the researcher and the children.

Researcher: This machine works with candies. If I put this candy in here, it will produce three candies out of its big side [*illustrates with the materials, A]*

Child 1: Triple. And if you put two candies in, it will make them six [*mentally*].

Researcher: How do you know this?

Child 1: Because it will repeat three two times [*illustrates with the materials, B*]

Child 4: No, it will repeat two three times. Because there are two candies [*illustrates with the materials, C*]

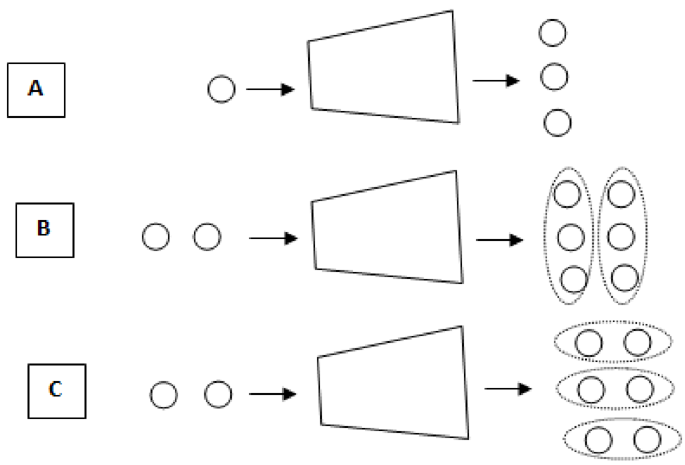

Figure 1: Introducing the 1:3/3:1 "fraction machine" (discrete quantity, multiple)

Child 1 discerned and verbalized the intended relation already with the first example (Figure 1, B), and offered an additional example. His answer, although numerically correct, did not model accurately the given situation. Child 1 presumably relied on one-to-many correspondence (one candy -> three candies, one + one candies -> three + three candies), a strategy that was not presented in the intervention. Child 4 recognized and corrected the “misstep” using the intended strategy (Figure 1, C).

In the second episode, the researcher had already introduced the 1:3/3:1 “fraction machine” for continuous quantities and the children had worked with tasks regarding the increase of the length of stick candies by a factor of 3. The researcher then asked about the inverse process:

Researcher: Now let’s see what happens if I put this stick candy into the big side of the machine. What do you think will come out from the small side?

Child 3: It will share it [*sic*].

Child 1: Yes, three times [*sic*]

Child 2: Where are the little pieces? [*tries with two smaller stick candies (1/2, 1/3) checking whether they fit three times into the given one*].

As this excerpt indicates, the children were able to anticipate what the machine would do, and also to use the intended strategy in order to find the outcome. However, none of them used the term “one third”. More generally, regarding submultiples, the children used spontaneously only the word “half” during the intervention. On the contrary, they adopted and used the terms for multiples (e.g., Child 1 in the first episode), and also attempted to generalize them. For example, Child 3 invented a word similar to “sixtuple” to refer to “the one that makes everything six times bigger”.

Quantity,	Pre-test					Post-test				
Relation	Ch1	Ch2	Ch3	Ch4	Total	Ch1	Ch2	Ch3	Ch4	Total
D, 1:2 (n=3)	1	3	1	1	6	2	2	2	3	9
D, 2:1 (n=3)	0	0	1	0	1	2	2	2	3	9
C, 1:2 (n=3)	0	1	2	1	4	1	2	1	3	7
C, 2:1 (n=3)	2	0	1	1	4	3	3	3	3	12
Total	3	4	5	3	15	8	9	8	12	37
D, 1:3 (n=2)	-	-	-	-	-	1	0	1	0	2
C, 3:1 (n=2)	-	-	-	-	-	0	1	0	1	2

Table 1: Total of correct responses per type of quantity, per relation, and per child in the pre- and post-test.

Performance

Children's correct and incorrect responses in the pre- and post-test were scored by 1 and 0, respectively. Table 1 presents the frequencies of correct responses in the trials corresponding to each relation (1:2, 2:1) across the tasks A, B, C, for discrete (D) and continuous (C) quantities. For the common part of the pre- and the post-test, this results in three responses per relation and per type of quantity; and similarly to two responses in the additional task (Dask D) of the post-test.

Table 1 shows that there was a considerable increase in correct responses after the intervention, at group as well as at individual level. For the common part of the pre- and post-test, the percentage of correct answers in the total of trials involving discrete quantities has increased from 29,2% to 75%; and the percentage of correct responses in the trials involving continuous quantities has increased from 33,3% to 79,2%. In Task D, each child responded correctly to one out of four trials.

Explanations

Children explanations during pre and post- test can be roughly categorized in two different types. The first type (Non-valid explanations) includes null explanations (e.g., "I don't know" or "I saw it"); pseudo-explanations that were relevant to the general context but not to its quantitative aspects (e.g., "Because he wants to eat chocolate after dinner"); and inadequate quantitative explanations, typically based on absolute quantity, rather than on quantitative relations (e.g., "because it's small", "because there are three").

The second type of explanations (Valid explanations) includes the cases where children expressed verbally and/or non-verbally (e.g., with gestures) a valid strategy that they used to make or verify their choice. We also included in this category explanations indicating that the children employed the principles underlying the situation at hand. In the following excerpts we present three examples of valid explanations, two in the context of continuous quantity, and one in the context of discrete quantity.

In the first example, Child 3 explained how he found half of a "chocolate bar":

Child 3: We cut in the middle and we got one half [*passes his hand over the "chocolate bar*]. And the other piece that remains is also half. It is the same as this one [*points to the correct alternative*]. Here, look! [*picks up the correct part and shows it fits two times into the "chocolate bar*].

In the second example, Child 2 was presented with a "chocolate bar" and was asked to find the one that was double (in length) than the given.

Child 2: Which chocolate bar should I choose for this small one? I know! That one! [*points to the correct alternative*]

Researcher: Why do you think it's this one?

Child 2: Because if you try to fill the big chocolate bar, you have to have two like the small one [*mentally*].

In the third example, Child 4 was told that the researcher gave some candies to "Helen and her little sister" and that the two girls shared the candies fairly. Then he was presented with the two candies that Helen took.

Researcher: Look, Helen took two.

Child 4: And her little sister another two [*mentally*].

Researcher: And how many candies did I have in the beginning?

Child 4: Four. And then they became two for each girl.

In this excerpt, Child 4 showed a quite principled understanding of the situation at hand: First, he appeared to employ the principle that in fair-sharing situations, the shares must be equal. He then used the two equal shares to compose the initial quantity, while also referring to the inverse process.

Table 2 presents the frequency of valid and non-valid explanations in the total of the trials in the common part of the pre- and the post-test (12 trials per test) and in the additional task of the post-test (4 trials), per child. There was a considerable increase in the number of valid explanations after the intervention, at group as well as at individual level. Overall, in the common part of the pre- and the post-test the percentage of valid explanations increased from 6,3% to 62,5%. In the additional task (task D) of the post-test, two of the children gave no valid explanations.

Tasks	Explanation type	Pre-test					Post-test				
		Ch1	Ch2	Ch3	Ch4	Total	Ch1	Ch2	Ch3	Ch4	Total
A, B, & C	Non-valid	12	11	12	10	45	4	5	7	2	18
	Valid	0	1	0	2	3	8	7	5	10	30
	Total	12	12	12	12	48	12	12	12	12	48
D	Non valid	-	-	-	-	-	3	4	2	4	13
	Valid	-	-	-	-	-	1	0	2	0	3
	Total						4	4	4	4	16

Table 2: Total numbers of valid and non-valid explanations in the pre- and post-test, per child.

CONCLUSION –DISCUSSION

We designed a program of activities introducing three fundamental multiplicative operations in a variety of situations, and terms for expressing multiplicative relations, across discrete and continuous quantity. We pilot-tested this program with a case study intervention, with four pre-primary children. The intervention was short and, arguably, very intense in terms of the amount of work required from the children.

Nevertheless, the results were promising. The program of activities was well within the children's range of abilities. By the 4th day, the children implemented the intended strategies; invented their own strategies; discerned and verbalized multiplicative relations and anticipated the outcome of multiplicative transformations. These competences were fairly stabilized for 1:2/2:1, as indicated by children's performance in the post-test in terms of correct answers and valid explanations. Children's ability to tackle relations beyond 1:2/ 2:1 was evident during the intervention, but did not reflect in their performance in the post-test. This is not an unexpected result, since 1:2/ 2:1 are more accessible to young children (Hunting & Davis, 1991) and the participants already had some relevant experience. The short duration of the intervention should also be taken into consideration.

A long-term, systematic intervention, with a larger and more diverse sample, is required to investigate whether a program of activities with the specific features can substantially enhance young children' multiplicative reasoning. In particular, it is worth investigating whether children who have acquired vocabulary relevant to multiplicative relations in early instruction (possibly, at kindergarten) and can use it in a variety of multiplicative situations are more competent in multiplicative reasoning in the long run (Vanluydt et al., 2021).

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