INTRODUCING MULTIPLES AND SUBMULTIPLES TO PRE-PRIMARY CHILDREN

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We present part of an ongoing topic-specific design research aiming at enhancing pre-primary students' multiplicative reasoning. We focus on an activity introducing vocabulary for multiples and submultiples with a view to enable children to express multiplicative relations verbally. We present the rationale and the design of the activity and findings of its first enactment with eight kindergarten children. The children appropriated the intended terms for multiples and, to a lesser extent, for submultiples as well, and used them to express multiplicative relations. The affordances and limitations of the activity are discussed, with a view to re-design it.

THEORETICAL FRAMEWORK

Research evidence indicates that there are early competences pertaining to multiplicative reasoning. Young children can trace multiplicative/proportional relations at a rudimentary level (McCrink & Spelke, 2016; Mix, Huttenlocher, & Levine, 2002); and can handle simple multiplicative situations involving discrete as well as continuous quantities (Hunting & Davis, 1991). These competences are limited; but may be enhanced if children are exposed to relevant informal and formal experiences (Hunting & Davis, 1991; Van den Heuvel-Panhuizen & Elia, 2020).

Such evidence has not been fully exploited in early childhood education even though learning objectives regarding multiplicative reasoning are included in mathematics curricula. For example, an analysis of a Greek mathematics curriculum (K-2) (Vamvakoussi & Kaldrimidou, 2018) has shown that learning objectives pertaining to additive reasoning were far more and were allocated far more teaching time, than the ones for multiplicative reasoning; and the latter referred far more to discrete, than to continuous, quantities. Moreover, the vocabulary necessary to express multiplicative relations was very limited. In kindergarten, in particular, all explicit learning objectives pertaining to multiplicative reasoning were placed in the context of discrete quantities, referring to the "equal groups" structure and the three problems that stem from this situation (corresponding to multiplication, partitive and quotitive division); and no term for multiplicative relations was mentioned, not even the word "half".

It is true that multiplicative situations that involve continuous quantities are more challenging. For example, in fair-sharing situations with discrete quantities, strategies like "dealing" one by one are accessible to young children. Such relatively simple strategies are not available for continuous quantities. Nevertheless, children appear to grasp underlying principles in multiplicative situations (e.g., "more recipients, smaller share") simultaneously for discrete and continuous quantities (Kornillaki & Nunes,

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2005). For different, albeit complementary, reasons, several researchers suggest that discrete and continuous quantities should be treated similarly. For example, Steffe (2013) argues that the construction of both types of quantity is based on the same mental operation, namely unitizing; and that counting schemes can be qualified as measuring schemes. Sophian (2004) argues that the role of the unit in counting is more similar to the role of unit in measuring than typically assumed. Vamvakoussi and Kaldrimidou (2018) stress that three fundamental multiplicative operations—iteration of a quantity, equi-partitioning, and measuring with different units—are the same for both types of quantity.

On the other hand, linguistic tools are important so that children can identify the same multiplicative relation in different contexts and be able to organize their formal or informal experiences of multiplicative situations (Hunting & Davis, 1991). Indeed, there is evidence that knowing simple words for multiples (e.g., “double”) in the first grade is associated with higher multiplicative/proportional reasoning competences in the second grade (Vanluydt, Supply, Verschaffel, & Van Dooren, 2021).

This paper presents part of an ongoing topic-specific design research (Gravemeijer & Prediger, 2019) aiming at developing a program of activities to support multiplicative reasoning at the first years of instruction. The three pillars of the program are a) addressing discrete and continuous quantities in a unified manner, b) providing experiences that pertain to three fundamental multiplicative operations (equi-partitioning, iteration of a unit and counting with various units (i.e., composite, fractional), and c) introducing terms for multiples and submultiples. The program has already been enacted once with kindergarten children (Pitta, Kaldrimidou, & Vamvakoussi, 2021), and has been redesigned, based on the findings. This paper focuses on a new activity intended as the introductory one in the current version of the program. This activity introduces vocabulary for multiples and submultiples with the purpose of describing multiplicative relations among the natural numbers 1-10—represented as continuous quantities—and the unit (1). The activity aims at capitalizing on children’s experiences with the sequence of natural numbers, as cardinal and ordinal numbers, and at grounding the intended multiplicative relations on measurement. Our local hypothesis (for this activity) was that the regularities underlying the natural number sequence as well as the production of Greek words for multiples and submultiples would support students to learn and produce such terms; and that multiplicative comparison via measurement would help them assign meaning to the terms.

It should be noted that Greek words for multiples are produced with a prefix that is the main part of the corresponding number word, and an invariant suffix (-“(a)plasio”). For example, “pente” is the Greek word for “five” – “pentaplasio” is the word for “quintuple”. Although there are slight variations in some of the terms, the regularity is quite salient. On the other hand, the words for submultiples are produced by the word for “one” (“ena”) and the corresponding ordinal number (similarly to English), also for

1/2 (unlike English). Although 1/2 can be verbalized as “one second” in Greek, a different word, similar to “half” (“miso”) is widely used.

METHOD

Design of the activity

The activity is story-based. The characters are imaginary creatures that play a sports game (two teams, “Pam’s team” and “Pom’s team”). The “players” are wooden cylinders with the same diameter but different height. Each team has a “captain” (Pam and Pom) representing the unit (4cm and 3cm high, respectively); and nine more players representing the numbers 2-10 with corresponding heights (multiples of the units). Each player has a paper “team shirt”, bearing the syllable “PAM” and a void to be filled by the players’ number (---PAM). Several copies of units are available.

A key part of the activity is naming the players: The 1st player is introduced as “Pam” and the children are asked to fill “the captain’s number” (1) in her shirt. The 2nd player is introduced as “Pam-Pam”, the 3rd as “Pam-Pam-Pam” and so on. The children are asked to a) to call the players by their names, clapping their hands with every “Pam”, b) write their numbers on their team shirts, and c) predict the name of subsequent players from the third one on. As the names become longer and more difficult to say, the nickname of each player is introduced which is synthesized by the part of the number words that serves as prefix for the corresponding multiple (hereafter symbolized by *n*) and the suffix “Pam” (*2*-Pam, *3*-Pam, etc.). Then children are asked to compare the players with Pam (e.g., “how many Pams does it take to measure *3*-Pam?”), using the available materials. The relationship is described symmetrically (e.g., “It takes 3 Pams to measure *3*-Pam”/ “Pam is one out of the three parts of *3*-Pam”. Then the terms for multiples and submultiples are also introduced symmetrically: The captain and the player verbalize their relationship (e.g., “I am the *3*-multiple of you” / “I am the one third of you”). In the process, the task for the children varies. One of the following is given and the rest are asked: a) the player as a physical object, b) the full name of the player, c) the players’ nickname, and d) the number on its team shirt. We note that if (a) is given, then children need to measure its height by the unit (length of Pam); and if (b), (c), or (d) is given, then children need to iterate the unit to find (a). A constant task across all variations is to find and express verbally the relation between the “player” and the “captain”, which is eventually explicitly asked using the word “relation”. The activity is repeated with “Pom’s team” (same multiplicative relations, different unit).

Participants

The participants were eight children (five girls), Greek native speakers with a mean age of 5 years and 9 months (ages varying from 5 years and 7 months to 6 years and 4 months). All children were students of a private Kindergarten in Ioannina. The children and their parents consented to their participation in the study.

Procedure

The children were tested via individual interviews before the intervention in order to investigate their prior knowledge on terms related to multiples and submultiples. They were familiar only with the term "half" but had no means available to explain or show what "half" means.

Our initial intention was to implement the entire program of activities, but this was not possible due to the pandemic. The introductory activity was implemented in May 2021, when schools opened after the lockdown. The children participated in groups (two groups, 4 members each) in the course of four weeks (one session per week for each group, about 45' each).

A retention test was conducted for every child individually, after the summer break (approximately four months later). The retention test was framed in the same context as the activity. Pam's team was presented to the child along with their team shirts, at random order. The researcher gave one player to the child who was asked to recognize the player and find her team shirt (Task A); verify/explain their answer (Task B); and express the multiplicative relation between the player's and the captain's heights (Task C). In total, 3 players were used—*2*-, *3*-, and *4*-Pam—addressing the relations 2:1/ 1:2, 3:1 /1:3, and 4:1/1:4, respectively; and corresponding to three trials for the tasks A, B, and C (A_k, B_k, and C_k, k=1, 2, 3).

RESULTS

During the intervention

The children indeed employed the natural number sequence to predict the name of the "next player" (e.g., Kostas below); they also started naming players beyond the given ones (e.g., Penny below):

> Kostas: Miss, I know who this is. It's *4*-Pam. You see, these are one-two-three [*pointing to the players already on the table*] and then comes four. So, this one is four.
>
> Penny: Let's get a really big one, *24*-Pam.

They also employed the recursive rule underlying the natural number sequence to predict the height of the "next player". For example, Aris selected the 5^{th} player and explained how:

> Aris: I counted. The previous was the fourth one, so now I need her plus another one. To reach the fifth one [*illustrates placing a copy of the unit on top of 4-Pam*]
>
> Researcher: How many Pams do you need to get from *5*-Pam to *6-Pam?*
>
> Aris: One. Each time we must put one like this [*pointing to a copy of the unit*] on top.

With the players' nicknames at hand, and after the introduction of the term for "double" (*2*-multiple), the children quite readily adjusted the players' nicknames to terms for multiples:

Researcher: We said that *2*-Pam is the *2*-multiple of Pam. Could you tell me what is the relation of *3*-Pam to Pam?

Penny- Kostas: She is the *3*-multiple! [*together*].

On the other hand, the terms for submultiples proved quite challenging. The main obstacle was the over-use of the word "half". For example:

Researcher: Now, could you tell me what is the relation of Pam to *3*-Pam?

Lora- Kostas: She is the one-half of her [t*ogether*]

Researcher: One half? But we needed three pieces to make *3*-Pam.

Penny: She is the three-half of her.

We reasoned that this obstacle was probably due to our decision to use the word "half" for the very first submultiple, although it is very different from the subsequent ones, because we knew it was more familiar to the children. In light of this realization, 1/2 was re-expressed as "one second", and the relevance of the sequence of ordinal numbers was brought to the children's attention. This was indeed helpful for the children; still, multiples remained easier to produce and use than submultiples, as illustrated in the following excerpt:

Researcher: So, what is the relation of *5*-Pam, the taller one, to Pam?

Aris: She is the *5*-multiple.

Researcher: And how about Pam? What's her relation to *5*-Pam?

Aris: One fourth.

Researcher: How many Pams do they fit in *5*-Pam?.

Soti: I know, I know! It's one fifth.

During the intervention the children gradually started to use the new terms more accurately, connecting them to the two fundamental operations (measurement and iteration of a quantity) to respond to the tasks and justify their answers. In the following examples, Lora verbalizes and explains the relation of *4*-Pam to Pam; Kostas gets the team shirt of *8*-Pam and explains how he is going to find this player and how she is related to Pam; and Nikos verbalizes and explains the relation of Pam to *3*-Pam:

Lora: *4*-Pam is the *4*-multiple of Pam because you need to stack four Pams to make *4*-Pam, Pam-Pam-Pam-Pam [*illustrating with the materials*].

Kostas: I'll build her first. I don't know how tall she's going to be. I need eight like these [*points to the copies of the unit, then stacks eight units*]. It's the *8*-multiple. It's eight Pams.

Nikos: We need three Pams to measure the big one. This one [*pointing to Pam*] is one out of the three pieces of that one [*pointing to 3-Pam*]. She is the one third of her.

Retention test

In task A, a response was coded as 1 if the child had recognized the player, otherwise as 0. In task B, a response was coded as 1 if the child had implemented a valid strategy to verify or explain her/his answer, otherwise as 0. In task C, we first examined whether the child used or not (coded as 1/0, respectively) the terms for multiples and submultiples to describe the intended relations. Then we examined how the terms were explained, or the relations described (in case the terms were not used).

In the case of multiples, we identified three types of explanation that were coded as follows: M1-Repeated Addition (e.g., "*3*-Pam is one Pam, and another one, and another one"); M2-Measurement (e.g., "This is *2*-Pam, because it contains 2 Pams" or "*2*-Pam is two Pams"); M3: Multiplicative comparison (e.g., "She is three times as Pam"). In the case of submultiples, three types of explanation were identified and coded as follows: S1- Part/Whole Relation (e.g., "She is half, because she is one of the other's two pieces"); S2-Measurement (e.g., "One fourth! Because you can fit four Pams into *4*-Pam"); S3 -Combination of S1 and S2 (e.g., "*2*-Pam contains two pieces, and one of them is half").

Table 1 presents the responses per child, across the three trials A_k, B_k, and $C_{k,}$ of the tasks, corresponding to the relations examined. $C_{k.1}$ and $C_{k2.}$ refer to multiples and submultiples, respectively; and are assigned two codes each, one for response, and one for type of explanation.

As can be noticed in Table 1, one child (Heleni) was unwilling to respond to any of the tasks; she related that she just wanted "to play with Pam's team". All the remaining children identified the given players (A_k); used a valid strategy (i.e., measuring or repeating a quantity) to verify their answers (B_k); and used all terms for multiples ($C_{k.1}$). In addition, all children explained adequately the terms for multiples ($C_{k.2}$). Two used systematically the same type of explanation (M1 for Kostas and M3 for Peny). With the exception of Soti, who never used M3 explanations, the remaining children moved from M1 or M2 to M3, for "bigger" multiples.

With respect to submultiples, there was more variation among the children. All used a term in $C_{1.2}$, and all preferred the word "half". Only three, however, explained their answer. Overall, four children used all terms correctly, but only two of them (Aris and Yianna) also explained all their responses (via S3). Nikos did not provide any explanation, while Peny explained "one third" and "one fourth", but not "half".

Lora used the terms "half" and "one third" but only explained the second term. Finally, Kostas and Soti did not use any term other than "half", which Kostas also explained. It is worth noting that these three children constructed their own terms for the ones they missed, mis-using the word "half". For example, Lora said "half of fours" instead of "one fourth".

Child	Relation: 1:2/ 2:1						Relation: 1:3/ 3:1						Relation: 1:4/ 4:1					
	A_1	B_1	C_1				A_2	B_2	C_2				A_3	B_3	C_3			
			$C_{1.1}$		$C_{1.2}$				$C_{2.1}$		$C_{2.2}$				$C_{3.1}$		$C_{3.2}$	
Nikos	1	1	1	M1	1	-	1	1	1	M3	1	-	1	1	1	M3	1	-
Lora	1	1	1	M1	1	-	1	1	1	M3	1	S2	1	1	1	M3	0	-
Kostas	1	1	1	M1	1	S1	1	1	1	M1	0	-	1	1	1	M1	0	-
Penny	1	1	1	M3	1	-	1	1	1	M3	1	S2	1	1	1	M3	1	S2
Aris	1	1	1	M1	1	S3	1	1	1	M3	1	S3	1	1	1	M3	1	S3
Soti	1	1	1	M1	1	-	1	1	1	M2	0	-	1	1	1	M2	0	-
Yianna	1	1	1	M2	1	S3	1	1	1	M3	1	S3	1	1	1	M3	1	S3
Heleni	0	0	0	-	0	-	0	0	0	-	0	-	0	0	0	-	0	-

Table 1: Responses and types of explanation per child in the retention test.

CONCLUSIONS-DISCUSSION

The first enactment of the sequence indicated that—as we expected—the children employed their knowledge end experiences of the sequence of natural numbers, as cardinal and as ordinal numbers; and they also discerned the regularities in the production of the Greek words for multiples to learn and produce such terms. The introduction of the terms for submultiples was challenging. We attributed the difficulty to our (rather unfortunate) decision to harvest the children's informal knowledge of the term "half" to introduce the very first term. This could explain the misuse of "half" in the production of subsequent terms, which persisted for some children up until the moment of the retention test. It has been suggested that the word "half" as well as the particular relation are privileged but may become an obstacle in the long run (Hunting & Davis, 1991). Using from the beginning and systematically the alternative term ("one second" in Greek) that is compatible with the subsequent ones may prove helpful. We had such indications during the intervention, and we will take them into consideration in re-designing the activity.

A promising finding is that the children appropriated the intended multiplicative operations and employed them to deal with the tasks, and to explain their answers. The retention test showed that these competences were retained for multiples, with the observation that "larger" multiples triggered more elaborate explanations, which will be take into consideration in re-designing the activity. On the other hand, in the case of submultiples, there were many limitations, and differences among the children, with respect to the use of terms and also the explanations. We note that the use of terms for submultiples did not necessarily imply that an explanation was provided; however, a relation was never described, if the appropriate term had not been used. This is an indication of the supporting role of relevant vocabulary for multiplicative reasoning, consistent with the findings by Vanluydt and colleagues (2021).

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