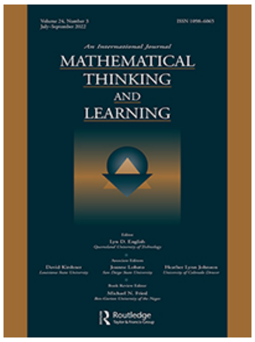

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Interpreting literal symbols in algebra under the effects of the natural number bias

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ABSTRACT

In this study, we investigated how secondary students interpret algebraic expressions that contain literal symbols to stand for variables. We hypothesized that the natural number bias (i.e., the tendency to over-rely on knowledge and experiences based on natural numbers) would affect students to think that the literal symbols stand for natural numbers only rather than for any rational or real number (integrity effect); and that the arithmetical values of the algebraic expressions are of the same sign as the expressions' phenomenal sign (phenomenal sign effect). The participants (138 8th and 9th graders) were asked to evaluate 48 statements about numbers that can or cannot be assigned to six algebraic expressions that contained literal symbols (e.g., a, -d-4). The results supported the main hypothesis of the study with respect to the integrity as well as the phenomenal sign effect and also indicated that the former was stronger than the latter. Additionally, the most salient characteristics of the form of each expression, such as its sign, appeared to affect students' responses regarding the arithmetical values they may represent. Theoretical and educational implications are discussed.

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Literal symbols; variables; algebraic symbols; natural number bias; negative sign

Introduction

Algebra has been described as the gatekeeper to upper-level mathematics and later achievement (J. L. Booth & Newton, 2012; Bush & Karp, 2013; Powell et al., 2019). Acquiring the concept of variable is widely recognized as a major challenge for students in the transition from arithmetic to algebra (Kieran, 2006; Lucariello et al., 2014; Schoenfeld & Arcavi, 1988). Students' understanding of variables influences their performance in many different algebraic topics such as solving equations and inequalities; representing functions; and problem-solving, via affecting students' solution strategies as well as the justifications they provide for their solutions (Knuth et al., 2005). Variables are usually represented by literal symbols (i.e., letters from the alphabet). There is common ground that capturing the variety of meanings and uses of the notion of variable in mathematics is very challenging (Schoenfeld & Arcavi, 1988; Wagner, 1983). The fact that letters are used in many ways in algebra, for example, to represent unknown numbers, generalized numbers, or functional relationships, is among the reasons for the many difficulties that appear on the part of the students (Asquith et al., 2007; Blanton et al., 2017; Brizuela, 2016; Kieran, 2006). At some point (at Grade 6 in Greece) variables are introduced in the mathematics curriculum as letters to stand for any number of a given set. This study focuses on two specific difficulties students appear to have with understanding variables (Christou, 2012; Christou & Vosniadou, 2009; Christou et al., 2007): a) the tendency to interpret literal symbols as natural numbers only (a phenomenon that we call the *integrity effect)*; and b) the

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tendency to assign to the arithmetical values of the algebraic expression an invariant sign, consistent with the one that the expression appears to have as a salient feature of its form (a phenomenon that we call the *phenomenal sign effect*).

In the present study we argue that these phenomena are effects of the natural number bias (i.e., the tendency to over-rely on knowledge and experiences based on natural numbers). We extend prior studies by employing a more systematic questionnaire that allows for investigating the following three questions, which haven't been answered so far: a) Which of the two natural number effects is stronger, the integrity effect or the phenomenal sign effect?, b) Does the type of the phenomenal sign of the expressions (positive or negative) makes a difference in students' judgments?, and c) Would students be more likely to assign non-natural values to an integer-like expression (e.g., k + 3) or to a fraction-like expression (e.g., 4/5y)?

Theoretical and empirical background

Students' difficulties with the use of literal symbols in algebra attracted the interest of mathematics education research from very early on. In a seminal study in the 80s, Kuchemann (1981) found that most 13-, 14-, and 15-year-old students interpreted literal symbols to stand only for *abbreviated names* of people or objects as, for example, a label for an object or an object itself (e.g., a to stand for apples and not for the number of apples), or as *coded numbers*, with values corresponding to their positions in the alphabet (L. R. Booth, 1988; Knuth et al., 2005; MacGregor & Stacey, 1997; Usiskin, 1988); fewer students considered them as *specific unknowns* (i.e., that they represent a specific unknown number with a fixed value); even fewer students appeared to consider literal symbols as *generalized numbers* (i.e., representing multiple values, one at a time), or *variables* (i.e., representing, at once, a range of numbers; Kuchemann, 1981). Subsequent studies further supported these findings and indicated that at some point, usually by Grade 10, students abandon the less sophisticated initial conceptions and come to develop an understanding of literal symbols as standing for specific unknown numbers (L. R. Booth, 1984; Knuth et al., 2005; Lucariello et al., 2014; MacGregor & Stacey, 1997; McNeil et al., 2010; Weinberg et al., 2016). However, understanding them as generalized numbers and as variables is much more difficult to accomplish, and such difficulties persist even when students receive detailed instructions to correct their mistakes (Asquith et al., 2007; Blanton et al., 2017; L. R. Booth, 1984).

Following the same line of research, recent studies that investigated students' misconceptions about variables in algebra also focused on different levels of understanding variables (i.e., whether variables are ignored, are understood as labels or as objects, are understood as specific or as generalized numbers; Lucariello et al., 2014; Powell et al., 2019). Christou and Vosniadou (2012) introduced a different question in this research area, which is the main research question also in this study: What kind of numbers are students willing to accept as substitutes for literal symbols representing variables? Are they willing to accept that variables may represent any real number?

At first glance, this question might seem narrow. One could argue that substituting numbers for literal symbols is an elementary school task, and that operating with variables independently of any referential meaning is the very essence of algebra. However associating variables with numbers, and thus, connecting the algebraic formalism with the "world of numbers" is a fundamental way of making meaning of the algebraic expressions, which also reflects on students' ability to operate with variables. As noted by prominent researchers who have studied students' learning of algebraic symbolism, even when students have developed a more abstract, structural understanding of an algebraic expression, which may allow them to perceive it as an entity in its own right, they do not necessarily disengage this entity from its initial reference to numbers, quantities, and to the relations among them that it represents (Arcavi, 1994; Kieran, 2007; Resnick, 1987; Schoenfeld, 2017; Sfard, 1995; Wagner, 1983). In fact, connecting algebraic expressions with their *referential* meaning – their *semantics*- helps students acquire a better understanding of the formal algebraic methods and increases performance in various mathematical tasks (Amado et al., 2019; Bush & Karp, 2013; Crowley et al., 1994; Demby, 1997; Graham & Thomas, 2000; Kieran, 1992; Resnick, 1989). For example, it may help someone understand why $2\times^2 < 0$ has no real number solutions, that $4 \times {}^4 > 0$ is valid for any real number, or convince someone that a + b = b + a. Although "guess and check" with specific numbers rarely provides mathematically accepted solutions it has often proven to be an effective problem-solving strategy (see for example, Johanning, 2004). For these reasons, limitations in the ways students associate literal symbols and numbers could result in distorted meanings of the algebraic expressions, which in turn may result in mistakes and low performance in tasks that include them, such as tasks related to equations, inequalities, and functions. For example, associating only natural numbers with literal symbols may result in accepting only one solution to equations such as $x^2 = 25$, which is a well-known phenomenon in the mathematics classroom.

There is evidence that when primary school students are first introduced to symbols or placeholders for symbolizing unknown numbers in equalities, they tend to interpret them as standing for natural numbers even when students are familiar with non-natural numbers such as fractions and decimal numbers (Switzer, 2018). It could be argued that these initial tendencies are not surprising, given the limited time students are exposed to non-natural numbers, compared to natural numbers. Moreover, literal symbols, as presented in junior high school mathematics textbooks (i.e., in worked-out examples as well as in tasks for students), typically stand for natural numbers, and less frequently for negative integers, or non-integer numbers (Dimitrakopoulou & Christou, 2018), even though the definition of variables that are included, specifically mention that any real number can be substituted for the variables in the algebraic expressions. Thus, it might be the case that students' initial tendencies to associate literal symbols only with natural numbers are strengthened and may persist at older ages.

Because of the strong connection between numbers and the concept of variable, Christou and Vosniadou (2012) embraced the framework theory approach to conceptual change, as applied in studying the development of the number concept (Vosniadou et al., 2008; Vosniadou & Verschaffel, 2004), to investigate students' interpretations of literal symbols representing variables that are contained in algebraic expressions. This theoretical framework assumes that before they are introduced to non-natural numbers, students have already consolidated a complex system of interrelated concepts and beliefs, tied around their knowledge and experiences with natural numbers, which are built from very early on. This initial framework theory for number underlies students' expectations about what a number is and how it is supposed to behave, shaping their interpretations of other kinds of numbers and affecting their reasoning and solution strategies in a variety of tasks (Vamvakoussi & Vosniadou, 2010; Vosniadou et al., 2008). The framework theory approach to conceptual change predicts that the transition from an initial concept of number as natural number, to an elaborate rational or real number concept, is a long-term process characterized by misconceptions, inconsistent responses (e.g., in similar tasks, albeit in different contexts), and also by *synthetic conceptions* of features and properties of numbers that reflect partial revisions of their initial understandings, in light of new information coming from instruction. So, for example, students are found to make systematic mistakes indicating that they draw on natural number knowledge when comparing fractions or decimal numbers (e.g., selecting the "larger" decimal as the bigger number; Nesher & Peled, 1986). They may also keep, for example, the initial idea that the size of a number depends on the number of its digits, combine it with the information that "thousandths are smaller than hundredths" and infer that "shorter decimals are larger," forming a synthetic conception (Van Hoof et al., 2017). Further, besides being more accurate, students are also found to respond faster to tasks that are compatible with natural number knowledge than to tasks that are not (Van Hoof et al., 2013, 2017). This phenomenon has been called *whole or natural number bias* ((Ni & Zhou, 2005; Van Dooren et al., 2015). We opt for the term *natural number bias* (hereafter, NNB).

In a series of studies, Christou and his colleagues (Christou & Vosniadou, 2009, 2012; Christou et al., 2007) tested the hypothesis that the NNB affects students' interpretations of algebraic expressions (i.e., combinations of numbers, letters, and signs of operations that include one or more variables, for example, a, k + 3, -b-1, x-2y). More specifically, it was hypothesized that students' initial conceptions of numbers, which are tied around natural numbers, would affect them to interpret literal symbols in algebra as symbols that stand for natural numbers, rather than for any real number. This is contrary to what they have been taught at school, and is also explicitly mentioned in their mathematical textbooks, namely that literal symbols stand for any real number, unless otherwise specified. This hypothesis was tested using different tasks, with open-ended and multiple-choice questionnaires, and with individual interviews.

Christou and Vosniadou (2005), Christou (2012)) administered to junior high school students a questionnaire with open-ended items, presenting algebraic expressions (e.g., a, k + 3, -b) and asking about possible arithmetical values that such expressions *could* and *could not* take. When responding to the affirmative question, students would typically give 1, 2, 3 as examples of possible values in the case of a; 4, 5, 6, . . . in the case of k + 3; and −1, −2, −3, . . . in the case of -b. When responding to the negative question, students would commonly give −1, −2, −3, . . . as examples of values that a *could not* take; and similarly, 1, 2, 3, . . . in the case of -b; and −2/-3, −3/-4 in the case of a/b. Such responses, which are correct in the first case but incorrect in the second, indicated that students tended to interpret literal symbols mainly as natural numbers. This finding was also corroborated via individual interviews (Christou & Vosniadou, 2012) with different participants. In those interviews the students claimed, for example, that 5d is always bigger than 4/d, supporting their claim via testing with natural numbers; explicit hints to test with at least one non-natural number were necessary for them to consider non-natural numbers as possible values for the literal symbols.

However, another possibility for *excluding* numbers as possible values of an algebraic expression was also traced. Christou et al. (2007) administered a questionnaire with multiple-choice items to secondary students. They were presented with algebraic expressions and asked to select from a given set of numbers that included positive and negative integers and non-integer numbers, those (if any) that they thought *could no*t be among the values of the given expression. A common pattern of responses indicated systematic exclusion of numbers that could not be obtained via substituting the literal symbols with natural numbers. An additional pattern, however, indicated systematic exclusion of positive numbers (either natural or non-natural), when the expression had a salient negative sign (e.g., -b), and vice versa. It should be noted that although it is possible for all the values of an algebraic expression to be of the same sign, algebraic expressions, in general, may have either positive or negative values, irrespective of the sign they appear to have. For example, -b is negative when b takes positive values, and positive when b takes negative values. The fact that signs attached to literal symbols do not always match their values has been long identified as a notable difference between literal symbols in algebra and numbers in arithmetic, which could create additional difficulty when the concept of variable is developed (Matz, 1979, as cited in Wagner, 1983). The studies reported above have offered empirical evidence in support of these theoretical assumptions. Further, this finding by Christou et al. (2007) has been corroborated by a replication study with Flemish students (Van Dooren et al., 2010) and is consistent with findings showing that students tend to associate the symbol of the negative sign with negativity (i.e., negative values; Vlassis, 2008), and the lack of the negative sign with positivity (Chiarugi et al., 1990).

There are two ways to explain this finding: First, it may be a direct byproduct of students' tendency to interpret literal symbols as natural numbers (i.e., a direct NNB effect). In this case, algebraic expressions such as -b or k + 3 would necessarily be considered negative and positive, respectively. Second, students may decide a priori that the expression has an invariant sign, based on the symbols "+" or "-" that the expression contains and are most salient to them. This could be a result of retaining (at least) one feature of the most elementary interpretation of literal symbols as natural numbers, that is, positivity. Thus, k + 3 could be allowed to take non-natural values, provided they are positive (e.g., 3.5); similarly, -b could be allowed to take non-natural values, provided they are negative (e.g., −0,5). Such interpretations can be accounted for as synthetic conceptions in terms of the framework theory approach to conceptual change (Christou & Vosniadou, 2012; Vamvakoussi et al., 2018; Van Dooren et al., 2010; Vosniadou et al., 2008), and can be also considered a NNB effect.

To summarize, there is some empirical evidence that the NNB has a dual effect on students' interpretations of algebraic expressions. First, it underlies students' tendency to interpret literal symbols as natural numbers only, a phenomenon that we call the *integrity effect*. Second, it underlies the tendency to assign to the arithmetical values of the algebraic expression an invariant sign, consistent with the one that the expression appears to have as a salient, albeit superficial, feature of its form. Thus, negative numbers may be excluded as possible values of positive-like algebraic expressions such as a, or k + 3; and positive numbers may be excluded as possible values of negative-like algebraic expressions such as -b. At the same time, values that cannot be derived via substituting literal symbols with natural numbers could be accepted, provided that their sign is consistent with the expected one. We call this tendency the *phenomenal sign effect*.

The present study

The purpose of the present study was to more systematically investigate the NNB effects on students' interpretations of algebraic expressions. We hypothesized that there would be a dual NNB effect on students' evaluations, namely the *integrity effect* and the *phenomenal sign effect*. Unlike previous studies (Christou & Vosniadou, 2005, 2012; Christou et al., 2007; Van Dooren et al., 2010) that asked students to assign values, or exclude numbers as possible values of algebraic expressions, in this study we opted for directly asking the participating students to judge whether a given number could be among the values of a given expression, or not. This allowed us to develop a more focused questionnaire, varying systematically the alternatives offered to students; and to test statistically which effect is stronger, the integrity or the phenomenal sign effect.

In addition, we investigated whether they would be differences in students' judgments due to the type of the phenomenal sign (positive or negative). Typically, there is no positive sign in the first term of the algebraic expressions (i.e., one writes k + 3 rather than +k + 3). On the contrary, the negative sign may appear in the first term of an algebraic expression (e.g., -k + 3). We reasoned that students would be more likely to infer a negative phenomenal sign in an expression such as -k-3, than they are to infer a positive phenomenal sign in an expression such as k + 3, because the phenomenal sign is more salient in the first than in the latter case.

Finally, we investigated whether students would be more likely to assign non-natural values to an integer-like expression (e.g., k + 3) or to a fraction-like expression (e.g., 4/5y). We reasoned that students who would accept a non-natural value in the first case could still assign only fractional values in the second, either because they fall back to interpreting literal symbols as natural numbers when the expression is more complex, or because the form of the expression steers them toward fractional values.

Method

Participants

The participants were 138 students from two Greek middle-class urban public high schools. Sixty-eight attended Grade 8, and 70 attended Grade 9; 77 were girls. All students were Greek native speakers, without special education needs. Greek students are introduced to positive rational numbers at Grade 3; to negative numbers as well as to the concept of variable at Grade 6; and to real numbers at the beginning of Grade 8. In particular, students by Grade 8 are intended to know that variables in algebra are represented by literal symbols, which stand for any real number unless otherwise specified. This is explicitly mentioned in the national mathematics curriculum and presented in the definitions that appear in the textbooks that are used in all Greek schools. Thus, our participants had in principle been exposed to instruction that would allow them to deal with the research tasks correctly.

Materials

We developed a questionnaire that presented 48 statements declaring either that it is possible or that it is not possible for a given algebraic expression to take a given numerical value. Students were asked to

Table 1. Examples of tasks by category of congruency/incongruency with integrity and phenomenal sign.

Task Category	Example
CongInt / CongSign	it is possible for -b to stand for −4 it is not possible for -b to stand for −8
CongInt / IncongSign	it is possible for -b to stand for 2 it is not possible for -b to stand for 3
IncongInt / CongSign	it is possible for -b to stand for −2.8 it is not possible for -b to stand for −1/2
IncongInt / IncongSign	it is possible for -b to stand for 4/3 it is not possible for -b to stand for 5/6

state whether they agree or disagree with each statement. Each of six algebraic expressions (*a, -b, k + 3, -d-4, 4/5y, −4/5z*) was combined with eight different numerical values (positive and negative integers, fractions, and decimal numbers) to produce the 48 statements. Half of the statements were given in affirmative form. An example of such an item is: "it is possible for *-b* to stand for the number 2." For this kind of task, the answer *I agree* was correct. The other half items were presented in a negative form as a means to also include tasks with *I disagree* as the correct response. An example of such an item is: "it is not possible for -b to stand for the number −8."

The above resulted in a design of tasks that were either congruent or incongruent with students' assumed tendencies. More specifically, there were statements a) congruent with integrity (CongInt) in which the given value was a natural number that could be derived if natural numbers were assigned to the literal symbol of the given expression (e.g., it is possible for *a* to stand for 6); b) incongruent with integrity (IncongInt) in which the given value was a (non-whole) decimal or fraction that could not be derived if natural numbers were assigned to the literal symbol of the given expression (e.g., it is possible for *a* to stand for 1/6); c) congruent with the phenomenal sign (CongSign) in which the sign of the given value was the same as the phenomenal sign of the given expression (e.g., it is possible for *a* to stand for 3/4); and d) incongruent with the phenomenal sign (IncongSign), in which the sign of the given value was the opposite of the phenomenal sign of the expression (e.g., it is possible for *a* to stand for −2). Thus, four main task categories were formed depending on whether each task was congruent or incongruent with the integrity and/or with the phenomenal sign (see examples in Table 1 and all the given tasks in the Appendix). For half of the congruent and incongruent items, the correct answer was "I agree" and for the other half it was "I disagree."

In order to test the effect of the form of the algebraic expression, four of the given expressions had an integer-like form (*a, -b, k + 3, -d-4*) and the other two had a fraction-like form (*4/5y, −4/5z*). Also, half of the given expressions had a positive-like form (*a, k + 3, 4/5y*), that is, a salient positive phenomenal sign and the other half had a negative-like form (*-b, -d-4, −4/5z*), that is, a salient negative phenomenal sign.

At the beginning of the questionnaire, there was a note that reminded students that literal symbols such as a, b, k, x, etc., are often used in mathematics to stand for numbers, and that this is the way they are used in this questionnaire. Also, they were asked to place an X in the box that best represented their answer (either agree or disagree).

Procedure

The questionnaire was initially tested in a pilot study with 16 participants from different public schools attending Grades 8 and 9. We explored whether the students could understand what the tasks asked them to do and whether the given instructions were helpful. Also, we measured the average time needed for the completion of the questionnaire. The results were used to finalize the form of the questionnaire and the procedure that was followed in the main study.

The revised questionnaires were administered by a researcher (second author) to the participants during their mathematics course, in the presence of their mathematics teacher, who was not informed

Table 2. Performance for each category of research tasks.

	Minimum	Maximum	Mean	S.E.
CongInt/ CongSign	3	12	8.38	0.19
CongInt/ IncongSign	0	12	6.01	0.21
IncongInt/ CongSign	1	12	6.11	0.19
IncongInt/ IncongSign	0	12	4.89	0.23

about the specific content or focus of the study in order not to prepare the students for this test. Their mathematics teachers only asked the students to complete the test with the necessary caution, as if this was a regular mathematics assessment test. All clarification questions by the students were answered by the researcher. The researcher, in the verbal instructions she gave at the beginning of the examination, read the written instruction to the students mentioning that the literal symbols in the given statements refer to real numbers, reminding the students that all numbers they know of are real numbers. The students had one teaching hour (40 minutes) to take and complete the questionnaires; however, none of the students exhausted the given time, as noted by the researcher, who was present throughout the examination. From her observations, most students completed the test within 20 minutes, and the questionnaires were collected immediately after they were completed.

Results

Students' responses to the tasks were scored on a right/wrong basis. There were very few items left unanswered; the corresponding missing data were treated as incorrect responses. The software suite SPSS 24 was used for the statistical analysis of the data. Overall the questionnaire showed sufficient reliability (*Cronbach's Alpha* = .738). The analysis of students' overall performance showed no main effect for grade [$F(1, 134) = 0.72$, $p = .398$], or school [$F(1, 134) = 0.997$, $p = .320$]. The mean scores were calculated for each category of tasks and are presented in Table 2. Students' highest performance appeared in the CongInt/CongSign tasks, and their lowest performance appeared in the IncongInt/IncongSign tasks.

The results of a two factor (integrity and phenomenal sign) analysis of variance with repeated measurement showed main effects for integrity [$F(1,137) = 106,36$, $p < .001$, $\eta_p^2 = .44$] and for phenomenal sign [$F(1,137) = 56,36$, $p < .001$, $\eta_p^2 = .29$] (normality and sphericity assumptions were satisfied). The effect size comparison showed that the size of the integrity effect was larger than the size of the phenomenal sign effect.

Further, for CongSign items, we compared students' performance in the integer-like expressions with their performance in the fraction-like expressions. The results of a t-test analysis showed significantly higher performance for the integer-like expressions than for the fraction-like expressions [$t(137) = 28.683$, $p < .001$] (see, Table 3).

Similarly, for CongInt items, we compared students' performance in positive-like expressions to their performance in negative-like expressions. A t-test analysis showed significantly higher performance in the first than in the latter [$t(137) = 4.127$, $p < .001$] (see, Table 3).

Table 3. Mean performance in integer vs fraction-like expressions, and positive vs negative-like expressions; error bars showing the 95% confidence interval.

Expression form	Minimum	Maximum	Mean	S.E.
Integer-like	4	16	10.31	0.23
Fraction -like	1	8	4.35	0.11
Positive – like	6	24	13.29	0.33
Negative – like	4	24	11.92	0.30

We also tested for potential differences due to the formulation of the question (affirmative/negative). A t-test showed no statistically significant difference between mean performance in the items corresponding to affirmative questions and in the ones corresponding to negative questions $t(137) = .368$, $p = .714$. Additionally, students' mean performance in the first 10 questions of the questionnaire was not significantly different from the mean performance in the last 10 questions of the questionnaire $t(137) = .121$, $p = .904$, indicating that students did not get tired or lost interest in completing the questionnaire.

Discussion

This study builds on, and extends, prior research on the effects of NNB on students' interpretations of algebraic expressions. NNB refers to a widely noted phenomenon, specifically that people tend to think primarily in terms of natural numbers in many different situations that involve numbers (Ni & Zhou, 2005; Van Hoof et al., 2013). Algebraic expressions contain literal symbols that represent numbers, which is precisely the reason why one could expect NNB effects on students' interpretations of algebraic expressions.

Previous studies (Christou & Vosniadou, 2009, 2012; Christou et al., 2007; Van Dooren et al., 2010) have provided evidence that students tend to misinterpret literal symbols to stand for natural numbers rather than for any (real) number (*integrity effect*). Students also tend to assign to the values of a given algebraic expression an invariant sign (*phenomenal sign effect*). As already discussed, both effects can be attributed to *NNB*.

In the current study, we looked at these effects in a more systematic way. We designed a research tool with tasks that directly asked students from 8th and 9th grade to agree or disagree with statements declaring that a specific number can or cannot be among the values of a given expression, varying systematically the kind of numbers and the form of the expressions.

The results showed that there is indeed a dual NNB effect on students' interpretations of algebraic expression, which was the main hypothesis of the study. Specifically, concerning the *integrity effect*, the students tended to perform significantly better in statements that were congruent with the belief that only natural numbers can be assigned to literal symbols (e.g., *k + 3 can stand for 9*) than in statements that were incongruent with this belief (e.g., *k + 3 can stand for 4/5*). For the *phenomenal sign effect of the NNB*, the students performed significantly better in the statements that were congruent with the belief that the numbers assigned to an algebraic expression should be of the same sign as its phenomenal sign (e.g., that *k + 3 cannot stand for −1*) than in the ones that were incongruent with this belief (e.g., that *k + 3 cannot stand for 5)*. These results provide further empirical support to previous findings in the field (Christou & Vosniadou, 2009, 2012; Christou et al., 2007; Van Dooren et al., 2010). The results also showed that the integrity effect is stronger than the phenomenal sign effect, which indicates that students are more willing to accept numbers with the opposite sign than the phenomenal sign of an expression as its values, than to accept non-integer number substitutions for the literal symbol of the expression.

Further, the results of this study highlight in more detail some other aspects of these NNB effects that have not been identified before. Specifically, in integrity-congruent statements, the students performed significantly higher when the algebraic expression had a positive phenomenal sign (e.g., *k + 3)*, than when the algebraic expression had a negative phenomenal sign (e.g., *-d-4)*. This finding indicates that the negative phenomenal sign exerts a stronger influence on students' interpretations of algebraic expressions than the positive phenomenal sign. This is consistent with studies showing that the symbol "-" is directly associated with negativity (Vlassis, 2008).

In addition, for the phenomenal sign–congruent statements, the students performed significantly better in integer-like expressions, than in fraction-like expressions. This finding indicates that students were more willing to accept that an integer-like expression could take non-integer values than to accept that a fraction-like expression could take non-fractional values. This may indicate that students who do consider non-integer values for the literal symbols in certain, relatively simple, cases, fail to do so in other cases, presumably because the form of the expression leads them to expect values that are similar in form. Alternatively, they may fall back on interpreting literal symbols as natural numbers when the algebraic expressions are more complex, as in the case of fractional algebraic expressions, where the variable is in the denominator.

Overall, the results indicate that students' interpretations of algebraic expressions are directly affected by their background assumptions of the possible values that literal symbols can take; and by their expectations regarding the possible values that the algebraic expression can take based on its form, which again depends on assumptions regarding the literal symbols. The most pervasive assumption seems to be that literal symbols represent positive integer numbers, that is, natural numbers. This is in fact a two-fold assumption consisting of the assumption of *integrity* and the assumption of *positivity*. These assumptions arguably make the expression k + 3 seem integer-like and positive-like; and the expression −4/5z seem fraction-like and negative-like. The assumptions of integrity and positivity apparently are not lifted at once and constrain students' interpretations of algebraic expressions, after students come to apprehend literal symbols as representations of numbers. Such intermediate states of understanding are predicted by the framework theory approach to conceptual change (Christou & Vosniadou, 2012; Van Dooren et al., 2010; Vosniadou et al., 2008) and may reflect synthetic conceptions when the assumptions are only partially lifted. For example, that k + 3 could be allowed to take non-natural values provided that they are positive; or k + 3 could be allowed to take negative and fractional values whereas -k-3 is allowed fractional but not positive values. This opens the possibility of intermediate states of understanding literal symbols as specific unknowns, as generalized numbers, as well as variables (Kuchemann, 1981), depending on the type of numbers that students expect the literal symbols to represent in each case. This offers a more nuanced picture of the development of understanding of literal symbols that is consistent with predictions stemming from the framework theory approach to conceptual change.

A limitation of this study is that the results are based on students' responses to a questionnaire, with 48 de-contextualized questions of the same type, half of which were negative. We included affirmative as well as negative questions so that "I agree" would not always be the correct answer because this would be incompatible with students' experiences with tests. We were aware, however, that negative questions increase cognitive load compared to affirmative questions (Evans, 1993). Thus, we tested for possible differences between students' responses in the affirmative questions and the negative questions and found no statistically significant differences. There is also the possibility that answering repeatedly to the same type of question might have been tiresome for some students who eventually might have lost interest. However, we didn't find significant differences in students' mean performance at the beginning and at the end of the questionnaire, an indication that students maintained their attention while responding to the given tasks. We cannot exclude, of course, the possibility that some students were not at all invested in responding to the tasks, even though their mathematics teacher was present and advised them to treat the questionnaire as a regular mathematics test. Future studies, in wider and more diverse populations also applying complementary methods, such as individual interviews or the use of contextualized tasks could shed more light on the phenomena tested here. This way, the results could be easier to generalize to wider populations.

Misinterpreting algebraic expressions do not come without consequences. The findings of Christou and Vosniadou (2009) study suggested that students' limited interpretations of the use of literal symbols in algebra caused them great difficulty in solving inequalities, and finding the domain of functions, especially when algebraic expressions appeared in square roots and in absolute values. Considering that algebra is deemed the gatekeeper of upper-level mathematics (Bush & Karp, 2013), it is reasonable to assume that narrow conceptions about the range of arithmetical values of algebraic expressions may restrain understanding and reduce performance in many mathematical content areas. This is because, as mentioned in the introduction, the referential meaning of algebraic expressions depends on the referential meaning of the literal symbols that they contain; and such meanings are essential for the development of students' more abstract understandings of algebraic expressions (see for example, Kieran, 2007; Resnick, 1987, 1989; Schoenfeld, 2017; Wagner, 1983).

Recent studies have provided empirical evidence that rational number knowledge act as a dominating contributor to algebra performance (Powell et al., 2019). In the same line, J. L Booth and Newton (2012) specifically stated that they "believe that there is something special about the relationship between fraction knowledge and algebra readiness" (p. 252), without, however, explaining what this relationship might be. From our perspective, one possible reason for this strong relationship could be the NNB-based constraints on students' interpretations of literal symbols as representations of numbers. Although associating literal symbols exclusively, or primarily, with natural numbers is perhaps an expected part of the developmental progress of the concept of variable, there is evidence showing that this tendency is robust. Indeed, it has been found in 10th graders (Christou & Vosniadou, 2009) and college students (Vamvakoussi et al., 2013). This does not mean that the students who show this tendency will deny that a variable can take on a negative or fractional value if asked explicitly; rather, evidence from individual interviews indicates that they do not tend to do it spontaneously and may only acknowledge this possibility after they are explicitly asked (Christou & Vosniadou, 2012). In fact, responding incorrectly, even though the knowledge necessary to respond correctly is available, is what makes NNB a "bias."

Several suggestions for tackling the NNB effects in algebra are already available in the literature. Some approaches aim at addressing already established misconceptions. One such suggestion is the use of erroneous examples as a means to make students aware of their misconceptions (Isotani et al., 2011). In the same vein, a teaching intervention that used refutational argumentation provided some promising results. Refutational argumentation consists of directly stating students' erroneous conceptions, for example, for the sign of algebraic expressions, and then refuting them by presenting students with alternative viewpoints. The results showed that students who attended the intervention did significantly better immediately after the intervention and those benefits were maintained in the retention test one month later (Christou, 2012). We note, however, that we do not expect one-time interventions to suffice; rather, systematic re-visiting of the same topics in different contexts is necessary, intending to support students to develop their metacognitive strategies (such as the "stop-and-think" strategy (Greer, 2009) as a means to monitor and control their thinking in such contexts. To this end mathematics teachers in the middle and earlier grades need to be aware of the particular difficulties, which is useful to actively analyze student work samples or classroom performance, or for setting the common mistakes out as counterexamples for students to confront, discuss, and analyze (Asquith et al., 2007; Bush & Karp, 2013). Mathematics teachers' awareness of such phenomena may be heightened by including these issues in programs for professional development. More importantly, such issues should be addressed in curricular materials for teachers.

On the other hand, some approaches may be followed longitudinally to alleviate the NNB effects. For example, teaching that emphasizes the connection between variables and real numbers could start from very early on, when students are introduced to symbols that stand for numbers in the context of equalities in arithmetic (Switzer, 2018). A functional approach to algebra, where letters represent varying quantities rather than unknown numbers, and algebraic expressions represent relations between quantities rather than a constant quantity (Kieran, 2006), could also be beneficial.

Such approaches could promote students' understanding of the concept of variable along with the concept of number in the long run, provided that students have the opportunity to work with a variety of non-natural measures of quantities. This is because an emerging understanding of variables as symbols to represent a range of numbers could promote an understanding of natural and non-natural numbers as unified systems of (rational or real) numbers, and vice versa. It should be noted that such approaches require a long-term perspective on mathematics instruction and the purposeful design of mathematics curricula.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Appendix: The tasks that were included in the questionnaire

It is possible for a stand for 6	It is not possible for -b to stand for −8
It is possible for -d-4 to stand for 3	It is not possible for -d-4 to stand for – 7/8
It is not possible for -b to stand for – 1/2	It is not possible for 4/5 y to stand for 16/5
It is not possible for k + 3 to stand for −1	It is possible for k + 3 to stand for −2.8
It is possible for 4/5 y to stand for −2	It is possible for -4/5 z to stand for −5.2
It is possible for – 4/5 z to stand for – 4/5	It is not possible for -b to stand for 5/6
It is not possible for k + 3 to stand for 5	It is possible for k + 3 to stand for −3
It is possible for 4/5 y to stand for 6.9	It is possible for -d-4 to stand for −9
It is not possible for -d-4 to stand for 5.6	It is possible for -b to stand for −2.8
It is not possible for a to stand for −5	It is not possible for -d-4 to stand for 8
It is possible for – 4/5 z to stand for – 4/5	It is not possible for a to stand for 1/6
It is possible for -d-4 to stand for −4.7	It is not possible for k + 3 to stand for 4.2
It is not possible for -b to stand for 3	It is not possible for a to stand for 3
It is not possible for a to stand for – 2/5	It is possible for 4/6 y to stand for −16/5
It is not possible for -d-4 to stand for −6	It is possible for -b to stand for 2
It is not possible for 4/5 y to stand for 12/40	It is not possible for – 4/5z to stand for 5.8
It is not possible for – 4/5 z to stand for 8/5	It is not possible for – 4/5 z to stand for – 8/5
It is possible for k + 3 to stand for 9	It is possible for a to stand for −8.9
It is not possible for 4/5 y to stand for – 12/5	It is not possible for k + 3 to stand for – 5/2
It is possible for – 4/5 to stand for 2	It is not possible for – 4/5 z to stand for −6.9
It is possible for -d-4 to stand for 4/5	It is possible for a to stand for 3/4
It is possible for k + 3 to stand for 4/5	It is not possible for 4/5 y to stand for −3.4
It is possible for a to stand for −2	It is possible for -b to stand for −4
It is possible for 4/5 y to stand for 8/5	It is possible for -b to stand for 4/3