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infinite sets; stressing that it is legitimate to wonder about infinity; emphasizing the relativity of mathematics, and strengthening students' confidence in the new definitions.

We evaluated the impact of traditional courses with little or no emphasis on students' intuitive tendencies to overgeneralize from finite to infinite sets, and of courses that were developed in line with the principles listed above, on high-school students and on prospective mathematics teachers' intuitive and formal knowledge of Cantorian Set Theory. Our findings in the different studies indicate that instruction that implemented these principles led to promote the reconstruction of knowledge structures (Tirosh, 1991; Tsamir, 1999). These interventions promoted the learners' awareness of the differences between finite and infinite systems, and of the contradictions that result from interchangeably applying different criteria when comparing infinite sets. Looking at these instructional interventions through different lenses could provide additional insights into their pros and cons.

In the Research Forum we shall show that the instructional design principles deriving from the conceptual change approach (as presented by Vosniadou et al., 2001) offer a valuable framework for analyzing and reflecting on instructional interventions in mathematics. More specifically, we focus on the mathematical notion of equivalency of infinite sets, using the instructional design recommendations of the conceptual change approach to analyze and reflect on related learning environments.

ASPECTS OF STUDENTS' UNDERSTANDING OF RATIONAL NUMBERS

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In this paper we examine students' understanding about the dense structure of rational numbers from a conceptual change perspective. We argue that students' difficulties reflect the fundamental presuppositions of their explanatory frameworks about number, which are tied around their understanding of natural numbers.

NATURAL VS. RATIONAL NUMBERS

Research in mathematics education has shown that prior knowledge of natural numbers gives rise to numerous misconceptions that pertain to both conceptual and operational aspects of rational numbers (e.g. Moss, 2005). Gelman (2000) argues that, even before instruction, children form an understanding about numbers, which is based on principles pertaining to the act of counting. This view is very close to a key assumption of the conceptual change theoretical framework that we adopt (Vosniadou, 1994; Vosniadou & Verschaffel, 2004), according to which children form initial explanatory frameworks about numbers, which are tied around their understanding of natural numbers. These frameworks facilitate thinking and learning about rational numbers, when new information is compatible with the underlying presuppositions, but cause difficulties and systematic errors when new information comes in contrast with what is already known.

In what follows, we will focus on students' understanding about the dense structure of the rational number set.

THE SET OF NATURAL VS. THE SET OF RATIONAL NUMBERS

The set of natural numbers consists of discrete elements which share a similar form, in the sense that any natural number is represented as the combination of a finite number of digits. Contrary to the natural numbers set, the set of rational numbers is dense, since between any two rational numbers there are infinitely many rational numbers. On the other hand, from a mathematical point of view, the set of rational numbers also consists of elements that share a common form in their symbolic representation, since any rational number is represented as the ratio of two integers. Alternatively, any rational number can be represented in decimal form, either as a simple or as a recurrent decimal. For the trained mathematician, it may be easy to move from one representation to the other, or even entertain both representations simultaneously, without losing the sense of the rational numbers set being a homogenous set, with dense structure. However, it is well documented that students have many difficulties moving flexibly and effectively among the various forms of rational numbers (e.g. Moss, 2005). We claim that students draw on symbolic notation to treat natural numbers, decimals and fractions as different, unrelated sorts of numbers. This claim is supported by evidence coming from research in various domains showing that novices tend to group objects on the basis of superficial characteristics (see for example Chi, Feltovich, & Glaser, 1981). Students' tendency to group numbers on the basis of their form may be enhanced by the fact that there are considerable differences between the operations, as well as the ordering of decimals and fractions.

We assume that the particular characteristics of the natural numbers set mentioned above (discreteness, elements with unique symbolic representation, homogeneity of forms) are key elements of students' initial "theories" about numbers and are bound to constrain students' understanding of the dense structure of the rational numbers set. Prior research has provided evidence that the idea of discreteness is indeed a barrier to the understanding of density for students at different levels of education (Malara, 2001; Merenluoto & Lehtinen, 2002; Tirosh, Fischbein, Graeber, & Wilson, 1999). In addition, Neumann (1998) reported that 7^{th} graders had difficulties accepting that there could be a fraction between two decimals, indicating their belief that decimals and fractions are unrelated sorts of numbers.

Based on the above remarks, we assume that the development of the concept of density requires conceptual change. We expect that students form synthetic models of the structure of rational numbers intervals, reflecting the constraints associated with their initial explanatory frameworks about numbers, as well as the assimilation of new knowledge into their incompatible knowledge structures.

These hypotheses were tested in two empirical studies with 9th and 11th graders (Vamvakoussi & Vosniadou, in press, 2004). In the second study, we also investigated the effect of the number line on students' responses to tasks regarding density. Following an ongoing discussion in the conceptual change literature about the effect of external representations (e.g. Vosniadou, Skopeliti, & Ikospentaki, 2005), we assumed that the effect of the number line is rather limited and may disappear, when the number line is withdrawn. According to our results, the presupposition of discreteness was strong in 9th grade and remained robust up to 11th grade. As expected, the presence of the number line did not facilitate students to a significant extent. Students' accounts of the rational numbers intervals reflected the expected constraints, as well as new knowledge and techniques pertaining to rational numbers –in this sense, they can be termed as *synthetic models*.

Adopting the conceptual change approach, we traced key elements of students' explanatory frameworks about numbers that may hinder further learning about rational numbers. We suggest that this approach could lead us, through a systematization of widespread findings on students' difficulties with rational numbers to a more clear picture of their explanatory frameworks about numbers and help to make detailed predictions about the barriers imposed by students' prior knowledge.

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CONCEPTUAL CHANGE IN THE NUMBER CONCEPT: DEALING WITH CONTINUITY AND LIMIT

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In our earlier studies 538 upper secondary level students (age 17-18 years) answered to a questionnaire concerning the density of numbers on the number line, explained the concepts of limit and continuity using their own words and estimated their certainty on their answers. 272 of these same students answered to the same questions half a year later. The results indicate that the majority of students based their answers on discrete numbers, everyday thinking of continuity and limit as a bound. In this presentation students' problems with these concepts is explained by the radical nature of conceptual change in the number concept, and by the dynamics of motivational and cognitive factors in conceptual change.

INTRODUCTION

Extending the number concept from natural numbers into the domains of more advanced numbers requires a radical change in prior thinking of numbers, a conceptual change. Research findings suggest that the majority of students have some kind of difficulties in this change. For example, in our large surveys 37 % of upper