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Conceptual Change: Philosophical, Historical, Psychological, and Educational Approaches

Programme

May 19-23, 2004
Delphi, Greece

Epistemological Beliefs of Primary School Students and Their Teachers and Their Effect on Conceptual Change

Florian Haerle, Carl von Ossietzky Universität, Oldenburg, Germany

B. Methodological Issues in Studying Conceptual Change

Chair: Lucia Mason, University of Padua, Italy

Discussant: Patricia Alexander, University of Maryland, USA

Transfer of Imagery and Runnability from Analogies to Models

John Clement, University of Massachusetts, Amherst, USA

Producing and Maintaining the Cognitive Order

Timothy Koschmann*and Alan Zemel, Southern Illinois University, USA

Intentional Analysis and the Interpretation of Interview Data

Liza Haglud and Ola Halldén*, Stockholm University, Sweden

Do Crucial Experiments Produce Conceptual Change? Theoretical and Educational Implications of Some Cases from the History of Science

Marcelo Leonardo Levinas, Universidad de Buenos Aires, Argentina, and Mario Carretero*, Universidad Autónoma de Madrid, Spain

Is conceptual change necessary for experts to change a learned skill error? The example of expert athletes in fin swimming with prior knowledge in the swimming style of butterfy

Maria Koulianou, Stella Vosniadou, University of Athens, Greece

C. Conceptual Change in Environmental Education

Chair: Lia Halkia, University of Athens, Greece

Discussant: Silvia Caravita, Instituto di Scienze e Technologie della Cognizione, CNR, Rome, Italy

Concept Formation in Environmental Education

Karolina Österlind, Stockholm University, Sweden

Conceptual Change: From Research to Instructive Practice. For a Timely Dealing with Students' 'Lamarckian' Views

Lucia Prinou*, Lia Halkia and Constantine Skordoulis, University of Athens, Greece

Effects of Conceptual Change Approach on Ninth Grade Students' Ecology Achievement: Attitudes towards Biology and Environment
Gülcan Çetin*, Hamide Ertepinar and Ömer Geban, Middle East Technical University, Turkey

Pupils' Conceptions as a Basis for Teaching Global Warming
Tiina Nevanpää, University of Jyväskylä, Finland

Changes in High School Students' Conceptions about Evolution by Natural Selection: A Case Study
Luli Stern* and Nirit Di Nur, Technion - IIT, Israel

14.30 - 17.00: Symposia

A. Conceptual Change in Mathematics: Theoretical Issues and Educational Applications

Organizers: Kaarina Merenluoto, University of Turku, Finland; Xanthi Vamvakoussi, University of Athens, Greece

Chair: Stella Vosniadou, University of Athens, Greece

Discussant: Anna Sfard, University of Haifa, Israel

An Application of the Conceptual Theory to the Comparison of Infinite Sets
Dina Tirosh and Pessia Tsamir, Tel Aviv University, Israel

Understanding Density: Presuppositions, Synthetic Models and the Effect of the Number Line
Xanthi Vamvakoussi* and Stella Vosniadou, University of Athens, Greece

Conceptual Change in Calculus: From the Circle's Tangent to a Curve's Tangent
Irene Biza*, Alkeos Souyoul and Theodossios Zachariades, University of Athens, Greece

Retrospective Interviews of Experts in Mathematics - Suggesting Conceptual Change
Kaarina Merenluoto, University of Turku, Finland

Tirosh, D., Fischbein, E., and Dor, E. (1985). The teaching of infinity. In L. Steefland (Ed.), *Proceedings of the 9th Conference of the International Group for the Psychology of Mathematics Education* (pp. 501-506). Utrecht, The Netherlands: University of Utrecht.

Tirosh, D. (1991). The role of students' intuitions of infinity in teaching the Cantorian theory. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 199-214). Dordrecht, The Netherlands: Kluwer.

Tsamir, P., and Tirosh, D. (1999). Consistency and Representations: The case of actual infinity. *Journal for Research in Mathematics Education*. 30, 213-219.

Tsamir, P. (1999). The transition from the comparison of finite sets to the comparison of infinite sets: Teaching prospective teachers. *Educational Studies in Mathematics*, 38, 209-234.

Tsamir, P. (2003). From "easy" to "difficult" or vice versa: The case of infinite sets. *Focus on Learning Problems in Mathematics*, 25, 1-16.

Vamvakoussi, X., & Vosniadou, S. (in press). Understanding the structure of the set of rational numbers: A conceptual change approach. *Learning and Instruction*.

Verschaffel, L., & Vosniadou, S. (in press). The conceptual change approach to mathematics learning and teaching. *Learning and Instruction*.

Vlassis, J. (in press). Making sense of the negative sign or becoming flexible in "negativity". Learning and Instruction.

Vosniadou, S. (1994). Capturing and modelling the process of conceptual change. In S. Vosniadou (Guest Editor) Special Issue on Conceptual Change. *Learning and Instruction*, 4, 45-69.

Vosniadou, S., Ioannnides, C., Dimitrakopoulou, A., & Papademetriou, E. (2001). Designing learning environments to promote conceptual change in science. *Learning and Instruction*, 11, 381-420.

Understanding Density: Presuppositions, Synthetic Models and the Effect of the Number Line

Xanthi Vamvakoussi* and Stella Vosniadou, University of Athens, Greece

Introduction

Already at a very young age, children have formed what has been characterized as "principled understanding" of natural numbers (Gelman, 2000), which is based on the act of counting. A basic characteristic of this initial "theory" of numbers is that numbers are discrete in nature. In fact, there is evidence that the property of discreteness of numbers may be neurobiologically based, in the sense that humans are prepared to learn and reason with natural numbers (Dehaene, 1998; Gelman, 2000). In the first years of mathematics education, children's informal knowledge about natural numbers is further confirmed and strengthened. There is plenty of evidence that, when new knowledge about rational numbers is to be acquired, prior knowledge about natural numbers may inhibit further understandings. These are the cases in which what is to be learned comes in conflict with what is already known. One such case is the development of the concept of density of rational numbers.

Understanding the structure of the set of rational numbers: A conceptual change approach

Contrary to the set of natural numbers, the set of rational numbers is dense: Between two successive natural numbers there is no other natural number, whereas between any two, non equal, rational numbers, there are infinitely many numbers. This is not in accordance with students' prior knowledge. Moreover, the idea of discreteness is deeply rooted into the human mind. In terms of the conceptual change theoretical framework proposed by Vosniadou (1994, 2001), discreteness is a fundamental presupposition of children's initial "theory" of numbers. In previous work (Vamvakoussi and Vosniadou, in press), we have assumed that understanding of density requires conceptual change. Qualitative data from a study with 16 ninth graders supported our hypothesis. According to our results,

- the majority of students generated errors that reflected the presupposition of discreteness. For example, students answered that "there is no other number between 0.005 and 0.006, because 0.006 comes immediately after 0.005" or that "there is only one number between 3/8 and 5/8, namely4/8".
- we diagnosed intermediate levels of understanding, reflecting students' efforts to assimilate new information about rational numbers, in their pre-existing structures of knowledge about natural numbers. For example, some students answered that "between 0,005 and 0,006 there are 0.0051, 0.0052, up to 0.0059". One student answered that there is finite number of numbers between decimals, but "if you turn them into fractions, then you find infinitely many numbers in between". Other students implied that between two fractions, there are infinitely many fractions. According to other students, different symbolic representations of the same number, e.g. 4/8, 8/16 etc. can count as infinitely many different numbers.

To summarize, the idea of discreteness seems to be a fundamental presupposition, which shapes students' understanding of the structure of the set of rational numbers. Yet, the development of the concept of density seems to be constrained by other parameters, too.

The empirical study

Based on the findings of the first study, we assumed that students' understanding of the structure of the set of rational numbers is also constrained:

- by their belief that different symbolic representations of the same number refer to different numbers.

- by their disposition to group numbers together on the basis of their symbolic representations.

To test these assumptions, we conducted an empirical study. We also aimed at investigating the effect of the number line on students' responses to questions about density. the participants of this study were 301 students, 164 ninth and 137 eleventh graders, all from schools in the Athens area. Based on the materials of the first study, we designed two types of questionnaires, a forced-choice questionnaire and one with open ended questions.

According to our results,

- the idea of discreteness is still strong in the case of eleventh graders.
- giving up the idea of discreteness is not an "all or nothing situation". For instance, the knowledge that there are infinitely many numbers between two decimals is not necessarily transferred to the case of fractions.
- students who answer that there are infinitely many numbers between two rational numbers, regardless of their representation, may still be constrained by the symbolic representation of the particular numbers. For instance, they may answer that "there are infinitely many fractions between two fractions.
- the belief that different symbolic representations of a number refer to different numbers is reflected in the answers of students of various levels of performance in the questionnaire.
- the effect of the number line on students' performance seems quite limited. Moreover, the effect "disappears", when the number line is taken away.

We will present these results and we will argue that students' misconceptions about density can be accounted for by the three assumptions mentioned above (the fundamental presupposition of discreteness, the belief that different symbolic representations of a number refer to different numbers and the disposition to group numbers of the same symbolic representation together). We will also argue that, students' misconceptions can be explained as synthetic models, since they reflect students' efforts to assimilate new information about rational numbers in their prior knowledge structures about natural numbers. Finally, we will attempt to explain why a widely used external representation of real numbers, namely the number line, seems to have a limited effect on students' understanding about density.

References

Dehaene, S. (1998). *The number sense: How the mind creates mathematics.* Harmondsworth Middlesex England: The Penguin Press. First published by Oxford University Press, 1997.

Gelman, R. (2000). The epigenesis of mathematical thinking. *Journal of applied developmental psychology, 21(1)*, 27-37.

Vamvakoussi, X. & Vosniadou, S. (in press).Understanding the structure of the set of rational numbers: A conceptual change approach. *Learning and Instruction.*

Vosniadou, S. (1994) Capturing and modelling the process of conceptual change. In S. Vosniadou (Guest Editor) *Conceptual Change. Special issue of Learning and Instruction*, 4, 45-69.

Vosniadou, S. (2001). On the Nature of Naïve Physics. In M. Limon and L. Mason (Eds.). *Reframing the Processes of Conceptual Change*. Kluwer Academic Publishers.

Conceptual Change in Calculus: From the Circle's Tangent to a Curve's Tangent

Irene Biza*, Alkeos Souyoul and Theodossios Zachariades, University of Athens, Greece

In this paper, we argue that a conceptual change approach could interpret some of the misconceptions dealing with the concepts of curve's tangent and derivative. In a pilot study, we make a case that students take for granted properties of circle's tangent in curves, where they do not apply in general. It seems that students generate resistant *synthetic models* to deal with tangent's problems.

Theoretical background – The aim of the study

Conceptual change studies the process of knowledge acquisition and especially in situations where the prior knowledge is incompatible with the new one. Consequently, various misconceptions occur in a not arbitrary way (Vosniadou, 1994). Many studies investigate conceptual change in the learning process of mathematical concepts (e.g. Merenluoto & Lehtinen, 2002; Stafilidou & Vosniadou, in press; Vamvakoussi & Vosniadou, 2002). In this paper, we present the results of a pilot study of a research concerning the development of notions of advanced mathematical thinking, from the conceptual change point of view. In particular, it deals with the concept of derivative and its geometrical representation, which is related to the notion of tangent line.

The notion of the tangent line appears in three stages during a student's schooldays. In geometry where students learn the tangent of the circle as a line, that has exactly one point, common with the circle. An intuitively obvious property of this line is that it divides the plane in two parts, one of which contains the whole circle. Later, the students are introduced to the conic sections. In these cases, the tangent's definition is different; it is more sophisticated. The "exactly one common point" property remains true in conics, but it is not enough to define the tangent. The "residence on one semi-plane" property is not valid in the case of the hyperbola but it remains true for each branch separately. Therefore, students do not change the previous intuitive images essentially. Finally, in a calculus course students encounter the concept of tangent to a point on a curve. Generally, none of the above properties remains valid. In fact, there are functions that have a tangent that not only does it have more than one intersection points with the curve, but it also splits the curve into two or more pieces (graph B6). At this level, a curve's tangent is defined through the concept of