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We present findings of the 3rd cycle of an ongoing design research study aimed at developing an instructional program to support pre-primary children's multiplicative reasoning. The 3rd enactment of the program was conducted with a class of 22 Year 1 and Year 2 kindergarten children (mean age 5 years and 7 months) who were pre- and post-tested individually. We focus on two tasks, common at both test moments, which were not addressed during the intervention. The results show that all Year 2, but very few Year 1 children were able to transfer what they had learned during the intervention to novel problem situations. Further, we found emerging inter-individual differences with respect to children's ability to represent mentally the quantities and operations involved in the given problems.

Keywords: multiplicative reasoning, kindergarten, multiplicative problems, performance, strategies.

Introduction

Research evidence indicates that young children have multiplicative reasoning competencies with respect to continuous as well as discrete quantities (Mix et al., 2002). These (limited) competencies can be enhanced when children are exposed to formal and informal multiplicative experiences (Hunting & Davis, 1991; Van den Heuvel-Panhuizen & Elia, 2020). However, early childhood education over-emphasizes counting and additive relations in the context of discrete quantities and pays less attention to multiplicative situations (Hunting & Davis, 1999; Sophian, 2004; Vamvakoussi & Kaldrimidou, 2018). This asymmetry has consequences for children's mathematical development. Indeed, children's performance in simple multiplicative tasks decreases when they are able to represent discrete quantities via counting (Ni & Zhou, 2005). Further, children miss the opportunity to build the conceptual background necessary for understanding important mathematical concepts, notably fractions (Sophian, 2004).

In this paper, we present results from the third cycle of a topic-specific design research study (Gravemeijer & Prediger, 2019) aimed at developing an instructional program to foster kindergarten children's multiplicative reasoning with respect to discerning and expressing multiplicative relations and solve multiplicative problems.

Theoretical background and design of the program

We based our design on three key principles: First, treating discrete and continuous quantities similarly and simultaneously, a choice grounded on different, albeit complementary, reasons: It has been argued that both types of quantity are constructed based on the same mental operation, namely unitizing (Steffe, 2013); that counting and measurement are more similar than typically assumed (Sophian, 2004); and that three fundamental multiplicative operations (namely, equi-partitioning, iteration of a quantity and counting/measuring with different units) are the same in both contexts

(Vamvakoussi & Kaldrimidou, 2018). Second, providing linguistic tools for expressing multiplicative relations, because such tools enable young children to recognize similar relations in different contexts and organize their formal and informal experiences with multiplicative situations (Hunting & Davis, 1991). This claim is supported by recent evidence indicating that simple vocabulary pertaining to multiplicative relations (e.g., "double") at Grade 1 is associated with higher proportional reasoning abilities at Grade 2 (Vanluydt et al., 2021). Third, highlighting the inverse relationship between the operation pertaining to multiplication (iterating a quantity) and those pertaining to division, particularly equipartitioning. This relationship is of great importance for multiplicative reasoning (Greer, 2012). Consistently with this idea, the relationships "A is a multiple of B" and "B is a submultiple of A" were treated as the same relationship seen from two perspectives.

These key principles were implemented in the design of the instructional program which consisted of three story-based activities (A1, A2, and A3) featuring a scenario with a sports team of 10 imaginary creatures. The characters were represented by wooden cylinders of the same base diameter and height ranging from 1 to 10 units in length. Multiple copies of the unit were available to students.

In the first activity (A1), terms for multiples and submultiples were introduced to describe the relationship between the "players" and the "team leader" (whose height was equal to the unit length). The relationship was established through measurement and expressed from two perspectives (e.g., A is triple B/ B is one third of A). The introduction of the terms capitalized on the salient regularity in their production in Greek (Pitta & Vamvakoussi, 2022) which is based on number words and was assumed to allow children to employ aspects of their knowledge of natural numbers productively (Steffe, 2013) to acquire vocabulary necessary to express multiplicative relations. The second activity (A2) targeted proportional problems: A discrete or continuous quantity was assigned to the "team leader" (i.e., the unit) and the quantity corresponding to a "player" was requested based on her height and vice versa. Finally, the third activity (A3) focused on multiplicative change using "fraction machines" (Hunting & Davis, 1991) which, unlike the typical ones, operated in two directions: on one side, the incoming quantity was multiplied by a natural number n, while on the other, by its reciprocal 1/n. This modification allowed for bringing to the children's attention the inversibility of the processes.

In all three activities, manipulatives made of cardboard were used to represent the quantities (disks for discrete, and bars for continuous quantities). Additionally, in A2 and A3 we used a manipulative representation of the ratio between the known and the unknown quantity. Figure 1 illustrates the process of finding a submultiple of a given continuous quantity (one third in the example) using this tool: Children are given options for the unknown quantity and are asked a) to estimate which is the appropriate one (1a), b) to check if the given quantity can be composed by iterating the selected bar 3 times (1b). The tool is then used to illustrate the partitioning of the initial bar (1c), as well as the quotient (1d). In the discrete condition, the available options were presented as linear arrangements of discs glued on paper strips. To find a multiple of a given quantity, the same tool was used in reverse. The use of the tool structures the intended processes similarly for continuous and discrete quantities, highlights the inverse relationship between multiplication and division, and was also assumed to help children keep track of important elements of the problem situation.

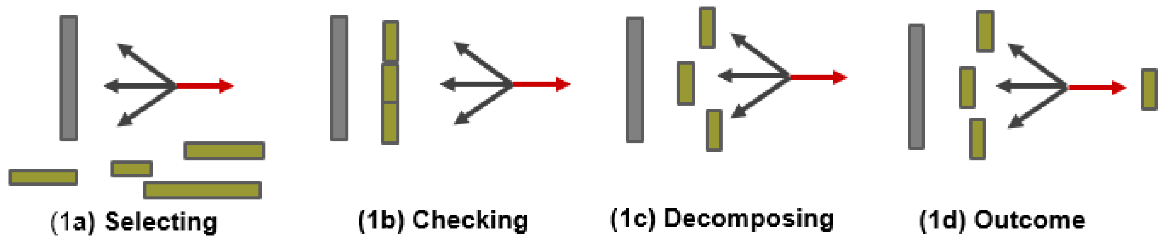

Figure 1: Finding one third of a continuous quantity with a manipulative representation of ratio

The program activities and materials have been designed, evaluated, and redesigned after two enactments with small groups of Year 2 kindergarten children (of a maximum of 4 members in each group) (Pitta, et al., 2021; Pitta & Vamvakoussi, 2022). It should be noted that Greek kindergarten classes typically comprise Year 1 (hereafter, Y1) children who enroll at 4 years of age; and Year 2 (hereafter, Y2) children who enroll at 5 years of age. Findings from the two enactments of the program showed that it was within the range of abilities of Y2 children and substantially enhanced their multiplicative reasoning competencies. In the 3rd cycle of the study, we enacted the program in a more ecologically valid setting: An entire mixed-age class (Y1 and Y2 children), participated in the intervention. We were interested in whether a) the program would have a positive effect on children's multiplicative reasoning competencies in the larger group/mixed age setting; b) the potential learning gains would be transferable to unfamiliar multiplicative situations; c) there would be inter-individual differences regarding potential learning gains. In this paper we focus on one type of difference, namely the extent of children's reliance on manipulative representations during problem-solving. This is because dependence on perceptual input can differentiate between children who solve a problem correctly, with less dependence indicating a higher level of mathematical thinking (Baroody, 2017).

Method

Participants

The participants were a class of 9 Y1 and 13 Y2 kindergarten children from an urban private school in Western Greece (mean age 5 years 7 months, ranging from 4 years 10 months to 6 years 4 months). Nine of the participants were girls (4 Y1). All children were native Greek speakers. They participated voluntarily with their parents' informed consent.

Procedure and research tools

The 3rd enactment started in January and was completed in May of the school year 2022-23. The students participated in the intervention in two groups of 11 mixed-age children; each group participated in nine sessions (approximately 45 minutes each) that took place in their classroom. During the intervention, each problem stemming from the activities was elaborated first for continuous, and immediately after for discrete quantities, to highlight the similarities. For any new problem, the teacher-researcher gave children the opportunity to solve it first on their own; she modelled the intended operations when necessary; she introduced the new terms, used them systematically, and prompted children to use them as well.

The children were pre- and post-tested via individual interviews. The interviews were conducted in two parts and were video recorded.

There were two categories of tasks used for pre- and post-testing. The first comprised problems similar to the instructional tasks, whereas the second comprised two tasks that had not been part of the intervention and tested for transfer of competencies to a new context. In the following, we will focus on these "transfer tasks". The first transfer task targeted "equal groups" and "equal measures" situations, in the discrete and continuous condition, respectively. A total of 18 problems was produced varying the number of equal groups or equal measures (n=2, 3, 4), as well as the type of problems that can be posed based on the same multiplicative situation. Figure 2, presents a model for this task in the discrete condition for n=3.The problems were formulated as follows:

Equi-partitioning (partitive division problem): Three friends (2a) will fair-share these candies (2b). Can you show me how many candies each child will get? How do you know?

Iteration of a quantity (multiplication problem): Three friends (2a) fair-shared a pack of candies. Helen took those two (2c). Can you tell me how many candies were in the pack? How do you know?

Measuring (quotitive division problem): A party of friends fair-shared these candies (2d). I don't know how many children there were (multiple copies of possible recipients available, 2e), but I know that Helen took those two (2c). Can you tell me how many children there were?

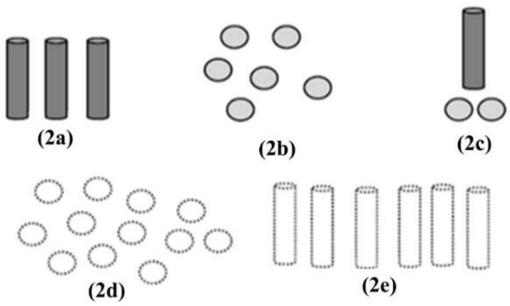

Figure 2: A model for the "equal groups" situation (n=3)

For the second task, the children were given discrete and continuous quantities and were explicitly asked to find a multiple and the corresponding submultiple of this quantity, for n=2,3,4. For example: "I have this chocolate (*paper strip bar*). I will give George one-third of the chocolate. Which of these pieces (*4 options*) do you think George will get? How do you know?" In this task (a total of 12 problems), recognizing and interpreting the terms was a prerequisite for solving the problems.

In all problems, the children were asked to explain their answers to ensure that their choices were not random. The ratio representation tool was available to the children.

Results

Performance

Children's response to each problem was evaluated and scored as 1 (correct) and 0 (incorrect), respectively. Each child's overall performance was calculated as the sum of their correct answers across all problems (a total of 30). Interestingly, among all possible total scores, only few were present in our data. Table 1 presents the frequency of each total score at pre- and post-test. At pre-test, only four children (including one Y1 child) solved two or three problems correctly. All correct answers were given in the discrete condition, specifically a) in a fair sharing problem with two recipients and, in one case, three recipients, b) recognition of the term "half," and c) recognition of the term "double".

Table 1: Frequency of total scores in the Pre- and Post-test

Pre-test			
Total score	Y2	Y1	Total
3	1	0	1
2	2	1	3
0	10	8	18
Total	13	9	22
Post-test			
Total score	Y2	Y1	Total
30	7	1	8
29	3	0	3
21	2	0	2
20	1	1	2
0	0	7	7
Total	13	9	22

At post-test, 7 out of 9 Y1 children had a total score of zero. Two of them refused to engage with the tasks from the beginning, while the others quitted midway without answering correctly even the simplest of tasks (i.e., when the relation in question was 1:2 / 2:1).

The remaining children (after the intervention) can be broadly divided into two groups: In the first (Group A), 11 children (including one Y1 child) solved at least 29 out of the 30 problems, while in the second (Group B), 4 children (including one Y1 child) solved 20-21 problems. It is worth noting that the incorrect responses were mainly observed in problems with $n>2$, primarily (but not exclusively) in the discrete condition, and mostly for submultiples.

Strategies

The children's strategies at the post-test were examined based on the extent to which they needed to use the manipulatives given and/or the ratio representation tool during the problem-solving process to represent the involved quantities or relationships. Three categories emerged:

The first category (S1, manipulatives-based strategy) included the cases in which a child used the given manipulatives, including the ratio representation tool, to represent all involved quantities. For example, in a partitive division problem (6 candies for 2 children), Child 1 used the materials representing the candies as well as the dolls representing the recipients; she initially matched each recipient with a pair of candies and then distributed the remaining candies one by one.

The second category (S2, partially mental strategy) included the cases in which a child used the materials, but not for all the involved quantities. For example, in the above problem, Child 2 used the material representing the candies, but not those representing the recipients. More specifically, she placed the 6 candies in a line, stated that she needed to create two groups of candies and pointed to the middle of the line saying, “There are two groups of three on the table, and Helen will take one group, and her sister will take the other one”.

The third category (S3, fully mental strategy) included cases in which a child solved the problem mentally, using the materials only to justify their answer. Such strategies indicate that the child performed mentally the necessary operations. In the following example, Child 3 employs an S3 strategy while solving a multiplication problem (3 candies per child x 3 children):

Child 3: [Looks at the available candies for a while] It's 9 candies [Arranges 9 candies in a line].

Interviewer: How do you know?

Child 3: [Points to a group of three with his fingers] Three, three, three makes nine. It's triple. Three times. Nine.

S3 strategies were also observed in the case of continuous quantities. In the following excerpt, Child 4 estimates in advance which of the options corresponds to half of the given quantity:

Child 4: [Looks at the options for a while and chooses the correct one] This is the one half.

Interviewer: How do you know?

Child 4: If you place it here [points at the bar], it is one of the two pieces. Look. [Opens thumb and index finger to the length of the piece] One, two [measures the chocolate].

An additional category (S0) included cases corresponding to incorrect or unjustified answers. Table 2 presents the frequency of these categories across all problems, for all 15 children at post-test. The dominant strategy was, by far, the S1 strategy; about 15% of the strategies were S2 or S3.

Table 2: Frequency and percent of different strategy categories, in the total of responses

Strategy Category	N (%)
S0 (No /Erroneous Strategy)	41 (9.1)
S1 (Manipulatives-based strategy)	342 (76)
S2 (Partially mental strategy)	37 (8.2)
S3 (Fully Mental strategy)	30 (6.7)
Total	450 (100)

We calculated the number of S2 and S3 strategies per child across all problems and we categorized the children accordingly, as presented in Table 3. Eleven out of 15 children demonstrated at least one S2 or S3 strategy. Four children (including one Y1 child) employed no such strategy; they all belonged to Group B. The remaining Group B children used at most 3 S2 or S3 strategies.

Table 3: Frequency of children groups based on the sum of partially (S2) and fully (S3) mental strategies across all problems at post-test

Sum of S2 and S3 strategies	0	1-3	4-6	7-9	10	14
Frequency	4	5	2	2	1	1

Finally, the sum of S2 and S3 strategies coming from Group A children ranged from 1 to 14. It is also worth noting that most of these strategies appeared in problems pertaining to the relations 1:2/ 2:1. However, 9 children demonstrated at least one such strategy for relations other than 1:2/2:1.

Conclusions and discussion

We presented findings from the third cycle of a topic-specific design research study (Gravemeijer, & Prediger, 2019) aimed at developing an instructional program to foster kindergarten children's multiplicative reasoning. In the previous two cycles, the program was enacted with small groups. In this cycle, an entire, mixed-aged, kindergarten class participated in the program in groups of eleven, which is a more ecologically valid setting in the Greek educational context. We presented findings for two tasks to which the children had not been exposed during the intervention and, thus, constituted a novel context for them. This claim was corroborated by findings indicating that at pre-test very few children were able to tackle a very limited number of the given problems.

Our results showed that, first, all Y2 children and two Y1 children had substantial learning gains: Not only did they recognize the terms for multiples (n=1, 2, 3, 4) and corresponding submultiples, but were also able to construct multiples and submultiples of given discrete as well as continuous quantities. Further, they were able to transfer what they had learned in the context of the intervention to the context of "equal groups" and "equal measures" situations, across all types of related problems (i.e., multiplication, partitive division, and quotitive division problems). The majority showed a remarkably consistent performance across problems; a smaller group of children had not yet stabilized their performance on problems involving relations other than 1:2/2:1 that are known to be the most accessible to children of this age (Hunting & Davis, 1991).

Second, it is evident that these research tasks were too demanding for most Y1 children. The two notable exceptions indicate that this is not merely due to age, highlighting the possibility of substantial inter-individual differences among Y1 children, beyond those between Y1 and Y2 students. We note that, based on our experience of Y1 children's participation in the intervention, we expected more of them to be able to tackle problems at the post-test moment, at least for the second task. Analysis of the intervention data will shed light on Y1 children's ways of engagement with the activities and inform our decision on mixed-age grouping in future enactments.

Third, there are indications of emergent inter-individual differences with respect to the extent manipulatives were necessary for children during the solution process. Mental representation of the problem situation differentiates children who solve correctly a given problem (Baroody, 2017), as is evidenced by the examples we presented. This feature will be further explored in combination with children's problem-solving methods (currently under analysis), with a view to outline a learning trajectory (Clements & Sarama, 2021) pertaining to early multiplicative reasoning.

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